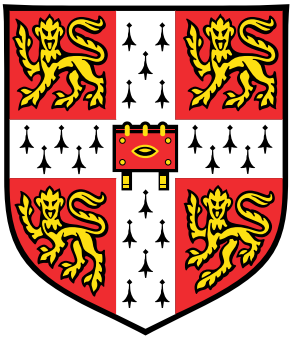
“Logic is a geometry of thinking”. Space and Spatial Frameworks in Wittgenstein’s Writings



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Declaration

I hereby declare that this dissertation is the result of my own work and includes nothing which is the outcome of work done in collaboration except as declared in the Preface and specified in the text. It is not substantially the same as any that I have submitted, or, is being concurrently submitted for a degree or diploma or other qualification at the University of Cambridge or any other University or similar institution except as declared in the Preface and specified in the text. I further state that no substantial part of my dissertation has already been submitted, or, is being concurrently submitted for any such degree, diploma or other qualification at the University of Cambridge or any other University or similar institution except as declared in the Preface and specified in the text. This dissertation contains less than 80,000 words including appendices and footnotes.

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Abstract

"Logic is a geometry of thinking".
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The thesis investigates the history and functions of space concepts in Wittgenstein's philosophy. It is based on a Kantian account which conceives of space not as a thing, but as an a priori framework which constitutes possibilities, not facts. The increasing abstraction and formalisation of geometry in the 19th century enabled Wittgenstein in his *Tractatus Logico-Philosophicus* to extend this formal account and to devise his concept of "logical space" as a universal and necessary manifold for all meaningful states-of-affairs. After his return to philosophy in 1929, he holds up the idea that necessity is not an extraordinary fact, but a feature of the logical framework which constitutes possibilities. Unlike in the *Tractatus*, however, he then speaks of spaces in the plural and highlights the differences between different "geometries" or "grammars". I emphasise the *plurality* of Wittgenstein's later space concept by presenting the various fields in which spatial terminology is used, as well as the *similarity* of these various instances by pointing out commonalities in the way in which they are used: the emphasis on possibility instead of truth, the distinction between "geometry" and "physics"(between logic and experience), but also the distinction between different kinds of geometries. These similarities allow me to recognise a number of concepts as closely connected to "space" – and thereby to one another – instead of highlighting their differences. Against views which argue for the complete disappearance of spaces and grammar in the late Wittgenstein's philosophy, I suggest that these concepts are not dismissed, but transformed after the middle period. The reasons for this transformation are the increasing importance of time, notably the change from static spaces to more dynamic frameworks, and the acknowledgement of empirical factors in logic: instead of an ontological separation of logic and experience it makes more sense to speak of different grammatical roles.

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Finally, I would like to warmly thank my family and friends for their support during the years of my PhD.

Abbreviations

NB Notebooks (Wittgenstein 1961)

TLP Tractatus Logico-Philosophicus (Wittgenstein 1922 and 1961)

SRLF Some Remarks on Logical Form (Wittgenstein 1929)

WVC Wittgenstein and the Vienna Circle (Wittgenstein 1979)

MN Moore's Notes from Wittgenstein's Lectures 1930–33 (Wittgenstein 2016)

VoW The Voices of Wittgenstein (Wittgenstein 2003)

PR Philosophical Remarks (Wittgenstein 1975)

BT The Big Typescript (Wittgenstein 2005)

BB Blue Book (Wittgenstein 1958)

PG Philosophical Grammar (Wittgenstein 1974)

LFM Wittgenstein's Lectures on the Foundations of Mathematics (Wittgenstein 1976)

RFM Remarks on the Foundations of Mathematics (Wittgenstein 1978)

PI Philosophical Investigations (Wittgenstein 2009)

PPF Philosophy of Psychology – A Fragment (Wittgenstein 2009)

OC On Certainty (Wittgenstein 1977)

CPR The Critique of Pure Reason (Kant 1999)

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Chapter 1

Introduction

Ich glaube einen Philosophen, einen der selbst denken kann, könnte es interessieren meine Noten zu lesen. Denn wenn ich auch nur selten in's Schwarze getroffen habe, so würde er doch erkennen, nach welchen Zielen ich unablässig geschossen habe.

Ludwig Wittgenstein
Nachlass, MS175

1.1 Why Space?

This thesis suggests a perspective on Wittgenstein's work as a whole, namely to look at his philosophy under the aspect of *space*. This requires some justification since Wittgenstein is not known as a philosopher of space nor does the concept seem to have much prominence in his major works, the *Tractatus Logico-Philosophicus* and the *Philosophical Investigations*. Moreover, it is a somewhat un-Wittgensteinian enterprise to suggest a single perspective on a thinker whose philosophical biography seems to be marked by significant shifts and who, at least in his later work, argued against any kind of monoperspectivism. Let me first say that my perspective does not amount to a claim that *everything* has an immediate connection to space, nor that, although I am largely concerned with philosophical *method*, there is one single method, a "spatial" one, which Wittgenstein follows in all his works. Rather, I present a plausible reading of the early and late Wittgenstein which accounts in a coherent way for the differences and similarities between the periods of his writing. The aim is indeed to find a certain overarching unity, but this unity is a unity of perspective: it does not exclude shifts in the development of Wittgenstein's philosophy – things can change under the aspect of

space – nor a pluralist understanding of the concept of "space" itself – like the family of game-concepts is central to his later work, his use of "space" can be important without all its instances having one thing in common. Indeed, if its function is understood, the concept can play its part even if the word itself does not appear at all. Understanding its role can be useful for the clarification of problems beyond the narrow scope of Wittgenstein scholarship with which this thesis is primarily concerned.

Why space? The motivation to read Wittgenstein under this aspect comes from a thorough study of the manuscripts from his so-called middle period or, to be more precise, from 1929 to 1933.[1] I am not the first to notice that in this period problems are strikingly often conceived in terms of space or geometry. Commenting on a typescript by Wittgenstein from 1930, Bertrand Russell names the two most important concepts in his thinking of that time: "He uses the words 'space' and 'grammar' in peculiar senses" (Russell 1968, 193). Most scholars who work on the middle period have in one way or the other taken account of this characteristic. It is, however, typically seen as a transitory phenomenon which vanishes after the early 1930s. Thus Joachim Schulte writes: "In my view, getting away from this mode of thinking in terms of different spaces [...] is a particularly important mark characteristic of the development of Wittgenstein's later philosophy". He especially contrasts thinking in spaces with "the motley of language-games we all know and love in the form it is presented in *Philosophical Investigations* and other writings by the later Wittgenstein" (Schulte 2006, 566).

Against this view I emphasise the continuity in Wittgenstein's thinking on which I shall elaborate a bit before explaining why space is particularly suited to illustrate it. Schulte uses the metaphor of a "motley" which is very popular among scholars who work on the *Investigations*. It is generally meant to highlight the difference between the early and late philosophy: while the *Tractatus* sought to find the "general form of the proposition", Wittgenstein later acknowledged "that what we call 'proposition', 'language', has not the formal unity that I imagined, but is a family of structures more or less akin to one another" (PI 2009, §108). The image of a motley can be illuminating in the sense that it captures this change as long as one does not dogmatically say "Language has the form of a motley", but rather "What we call language is not *one* thing, but a motley of forms".

It would, however, be misleading to call the later Wittgenstein's writings themselves a "motley of language-games". According to the OED a "motley" is an "incongruous, multifarious, or confused mixture or assembly". To be sure, the style of the *Investigations* is very different from conventional academic style and may make the impression of lacking

1. One can speak of middle period until 1937 when the *Philosophical Investigations* start to take shape, but the early middle period can be treated separately as I shall show below.

a coherent structure.[2] But at the same time they are a very carefully arranged text which tries to account for a number of inherently connected problems. To deny this was surely not Schulte's intention: after all, he edited a "critical-genetic" edition of the *Investigations* which accurately presents them as the result of a long and painstaking process of writing, selecting, revising, and rearranging philosophical remarks over the course of several years.[3] But by stressing the difference between space and motley, he draws a stark contrast between Wittgenstein's systematic, spatial, middle period and the more motley late period. I propose a different emphasis. The *Investigations*, despite their loose structure, do suggest a nexus of problems with which they are concerned. When Wittgenstein compares his remarks to sketches made during a journey through a "field of thought", he says that the "same or almost the same points were always being approached afresh from different directions, and new sketches made" (PI 2009, 3e). Reading these sketches together with the manuscripts from the middle period, it struck me that the problems that were accounted for in terms of space had not disappeared, but were now put in terms of different concepts that inherited important features of what "space" stood for in the 1930s. To be sure, this comes along with significant shifts, which shall be discussed in this thesis, but there is also a sense in which the problems stay the same. The preface to the *Investigations* was written in 1945. It states that the thoughts expressed therein had occupied Wittgenstein "for the last sixteen years" and does not mention any major break in this period. It even suggests a certain continuity with the *Tractatus*, albeit one which involves contrast:

> Four years ago, however, I had occasion to reread my first book (the *Tractatus Logico-Philosophicus*) and to explain its ideas. Then it suddenly seemed to me that I should publish those old ideas and the new ones together: that the latter could be seen in the right light only by contrast with and against the background of my older way of thinking.

If the *Tractatus* and the *Investigations* were indeed published together, there would be a kind of link between them, namely the motto by Johann Nepomuk Nestroy: "The trouble about progress is that it always looks much greater than it really is". Conversely, this might suggest that the continuity is greater than it seems, a continuity which is blurred by overemphasising differences between the *Investigations* and the earlier works.

The difficulty in talking about these things is to maintain the balance between continuity and discontinuity without overly emphasising one of them to fit a narrative. Peter Hacker has

2. And this may be said *a fortiori* about his unpublished manuscripts which likewise lack a hierarchical structure, but are composed in the form of "remarks", short paragraphs separated by blank lines.

3. It has been published in German in 2001. For the latest English version, which I shall use here, Schulte contributed to the translation and to the introduction in which the genesis of the text is sketched (PI 2009).

advanced an influential reading which sees only few refinements on the way from middle to late Wittgenstein. Especially the grammatical method, which is assumed to be central in both periods, is said to have not changed since the mid-thirties. "It is patent that the conception of philosophy he advanced in the *Investigations* is, in all respects pertinent to his grammatical investigations, perfectly consistent with that proposed in *The Big Typescript*" (Hacker 2012b, 17). This view has been criticised for relying too much on sources of the middle period and for failing to recognise the changes the concept of "grammar" underwent in the late 1930s.[4]

To outline my claim of continuity, I shall in this introduction critically discuss two approaches that I am sympathetic to insofar as they try to provide an account for these changes from middle to late Wittgenstein. What I do not agree with is the exaggeration of the differences and the tendency to make the progress "look greater than it really is". The first is Mauro Engelmann's interpretation which holds, against Hacker's view, that "[n]othing of the 'old grammar' is in place in the PI" (Engelmann 2013b, 261).

In his account of Wittgenstein's development, Engelmann is concerned with differences and indeed draws them so sharply that he suggests speaking of "different philosophies", not just the traditional Wittgensteins I and II, but several philosophies after 1929. He divides these philosophies as follows: after the *Tractatus* and the following period with the well-known biographical stages (prisoner of war, primary school teacher, architect) Wittgenstein engaged with a modification of his early work by looking for a "phenomenological language" which was supposed to translate immediate experience into a unified notation. From 1930 this project was no longer pursued. Instead, Wittgenstein now stressed the role of "grammar" which stands for a comprehensive system of rules governing language. This, however, was gradually replaced by what Engelmann calls the "genetic method", that is, a method that tries to comprehensibly present how a certain problem has come about, often by telling a story about how the relevant words have been learned and how certain habits have emerged. Because of this latter feature the genetic method goes hand in hand with an "anthropological view", a position that tries to grasp things more objectively by treating familiar modes of acting and speaking like an anthropologist might regard a foreign culture (cf. Engelmann 2013b, 1–4).

The general outline of this narrative is certainly plausible. There are indeed shifts in Wittgenstein's writing that follow the steps mentioned by Engelmann: from phenomenology to grammar to something close to anthropology as a tool of the genetic or "therapeutic" method. But, as Alois Pichler has pointed out in his review of Engelmann's book, other

4. In the same paper (2012b), Hacker lists instances of grammar from the middle and late period in order to show that nothing has changed . While this claim is largely correct for the aspects considered, the list itself is not complete and he does not consider changes of context which may alter the meaning of a remark significantly (cf. Engelmann 2013b; Dobler 2013; Uffelmann 2018).

stories are possible than this somewhat teleological account which sees each new step as replacing the previous until a final level is reached. Even though Engelmann does acknowledge that seeds of the genetic method are already present in the *Big Typescript*, his presentation follows largely a "'not yet there, but coming next' scheme" (Pichler 2016, 7). However, there are good reasons to consider Wittgenstein's development "more in terms of a continuous struggle between constantly present views and approaches and the iterative weighting of their relation and less in terms of a linear development from one view to another" (8). Pichler hints at the possibility of several simultaneous positions that Wittgenstein may have found attractive at a given time. Indeed his (like David Stern's) account stresses this "polyphonic" nature of Wittgenstein's style not only in the middle period, but especially in the *Investigations*. While Engelmann argues that vestiges of the "old grammar" are only quoted by the late Wittgenstein in order to be criticised, one can also regard them as voices in their own right which strive for clear structures and systems, thereby acting as a counterbalance to the negative and anti-systematic voices in the *Investigations* to which Engelmann seems to grant final authority (cf. Pichler 2004; Stern 2017).

Here is a characteristic example from Engelmann's book: drawing on George Edward Moore's report about Wittgenstein hinting at the vagueness of concepts like "proposition" and "sense" (a step away from the *Tractatus* made in 1932–33), he concludes: "The lack of sharp limits to 'proposition' and 'language' thus suggests that the idea of 'grammar' as the rules of sense is idle. Tabulating those rules is useless, if one is looking for the limits of language and the limits of sense" (Engelmann 2013b, 160).

But this is not what is said in Moore's notes, instead they say: "although it is not sharply bounded, the expression 'makes sense' is useful as 'game' is useful" (quoted by Engelmann on the same page).[5] This is evocative of Wittgenstein's later remark about the concept of "game" as a family of interrelated concepts (cf. PI, §§ 66–67), but also of him questioning the statement that vague rules have no function at all: "'Still, it isn't a game at all, if there is some vagueness *in the rules*.' But is it really not a game then?" (PI, §100). The point of this latter remark seems to be that a game with vague rules is still called a game; that an enclosure with a hole is still an enclosure; that we are misled by an ideal conception of game or enclosure which prevents us from seeing that these words work well without being everywhere defined with absolute rigidity. Likewise, a concept of proposition or grammar that allows vagueness

5. I cannot completely coordinate the passage from this edition (1965), in which Moore summarised his notes into essays, with the recent edition of his notes (2016); but in February 1933, Moore reports Wittgenstein expressing himself very similarly: "We are quite right to use the word 'game', so long as we don't pretend to have drawn a definite outline [...] 'Sense of a proposition is its use' – & this, of course is vague". Immediately after this, however, Wittgenstein says something that clearly contradicts Engelmann's reading, namely an affirmative evocation of his "old grammar": "When we do philosophy, we give rules of grammar wherever there is a philosophical difficulty" (2016, 280–81).

can still be useful to decide whether something makes sense or not. I do not agree with Engelmann when he states that the "whole project of drawing the limits of sense, which was the original purpose of Wittgenstein's notion of 'grammar' since PR, is abandoned" (2013, 171) and replaced by describing the use of words from an anthropological perspective. There is a certain tension between these two methods, which shall be discussed in the later chapters of this thesis, but I shall also provide evidence that the grammatical method of sense-constitution does not disappear completely after the mid-thirties. The strategy to prove this continuity shall be to carve out the function of "space" and "grammar" in the middle period and to compare it to the arguments in the late period. What motivates Engelmann's sharp distinction is, among other things, his assumption that the middle Wittgenstein's "grammar" is a comprehensive system which accounts for the totality of language. It is a popular misunderstanding that "grammar" was supposed to be an actual book in which rules for a totality of uses are stated. A possible source of this confusion is Wittgenstein's analogy of grammar as an "account book" of language which, he claims, should feature a certain "completeness" (cf. TS213, 526r; Engelmann 2013b, 139; Uffelmann 2018, 140). However, in the manuscripts it becomes clear that this metaphor is introduced in order to stress a different sort of completeness, namely that grammar need not be concerned with emotional concomitants of utterances. The feelings of a businessman play no role in an account book which only states transactions (not rules) just like in Wittgenstein's grammatical descriptions the feelings of the speakers should not make a difference. In the sense in which the lack of feelings does not constitute a gap in the account book, grammar is complete. It is not complete in the sense of a totality of transactions or legitimate uses of a word respectively – this would be absurd except for a limited period of time, but what would be the purpose of such a grammar (except for historical linguistics perhaps)?

At this point it makes sense to bring in the second foil to my approach, namely James Conant's ideas about Wittgenstein's method. Although known as a champion of the "resolute reading", which tends to emphasise similarities between *Tractatus* and *Investigations*, he has suggested a rather strong division between middle and late Wittgenstein in terms of a transition from *one* philosophical method to a plurality of methods.

His paper, too, is based on lecture notes which are a fairly reliable source of Wittgenstein's words.[6] However, from Wittgenstein's diary we know that these words need to be handled with care. In 1936, he admits retrospectively that he often pretended to his students to be certain of ideas when he was actually still in doubt about them (cf. MS183, 145).

6. He refers to Desmond Lee's notes; these largely correspond to those by G.E. Moore who is probably the most reliable source. Generally however, the notes by students can be problematic. Alice Ambrose's notes, for example, have been shown to include some significant misunderstandings (cf. Schulte 2006, 564; Nedo 2012, 321).

Nevertheless, one statement from those notes has become very famous and is used by Conant to support his argument: it is the bold claim that philosophy will be transformed ("as when chemistry developed out if alchemy") because "a method has been found" and instead of great philosophers now "there can be *skilful* ones" (MN 2016, 67). This stands in obvious contrast to another, later, remark which has also become very prominent despite its philologically precarious status: "There is not a single philosophical method, though there are indeed methods, different therapies, as it were". It is written on a slip of paper that was added to the final typescript of the *Philosophical Investigations* and is generally considered an addendum to §133, although there are scholars who question this attribution.[7]

From these two remarks Conant builds an intriguing narrative about Wittgenstein's development. Unlike Engelmann, he does recognise that grammar was not to be comprehensive, but that there are indeed "grammars" and that this is a first step away from the universalism of the *Tractatus*. However, there still remained the universalism of method as expressed in the lecture note. So Conant suggests a first transition from *one* logic to multiple grammars and a second transition from *one* method to multiple methods. Between these two transitions lies the middle Wittgenstein, roughly from 1929 to 1937. Conant sees that space plays a role in this development, yet he strongly believes the concept of space to be universal, as it actually was in the *Tractatus*; and this prevents him from seeing that what he calls "grammars" is connected to "spaces" in the middle period:

> For the problems of philosophy no longer rest for Middle Wittgenstein on a misunderstanding of something we can call *the* logic of our language, where it is crucial to the point of the definite article here that there is just that one logical space. (That is the point of the spatial metaphor, after all; as Kant almost says: all parts of space must be parts of *one* space.)[8]
>
> [...] Early Wittgenstein's conception of *the* logic of our language gives way to Middle Wittgenstein's conception of grammars, where the emphasis on the plural now becomes essential to the conception. Starting in the middle period, an interest prevails in mapping the contours of alternative logical terrains which cannot be accommodated within a single space. (Conant 2011, 640)

This has, I believe, an advantage over Engelmann's view, but overemphasises the transition from method to methods on the basis of two rather marginal remarks. It is by no means clear

7. Among them is Engelmann who insists on the primacy of his "genetic method". He also suggests that the later Wittgenstein's method can indeed be said to be *one*, but that it is inherently open. With this I agree, but I would hold that it should also be open to the grammatical, spatial, methods of the middle period and not nullify them.

8. The "almost" Kantian idea of a singular space and its reflection in Tractarian logical space shall be addressed in chapters 2 and 3 of this thesis.

what the single method was supposed to be.[9] Shortly before Wittgenstein gave the lecture on method to Moore and his other students in October 1930, he wrote in his diary that he had not thought about philosophy for three weeks and worried about not knowing what to say (cf. MS183, 46; 8 October 1930). Three weeks earlier he had written in a manuscript volume: "the method of philosophy is to listen to *all* voices and to reconcile them all" (MS109, 159; 19 September 1930). Of course, Wittgenstein speaks of method in the singular here, but is it really justified to make a stark distinction between method and methods if the method is supposed to account for *all* voices, that is for a plurality? Perhaps the idea of reconciliation can be read in Conant's way, but in Wittgenstein's practice, in his manuscripts from that time, he clearly tries to follow the pluralist part of his programme and listen to a number of different voices which he frames in terms of different grammatical spaces. These spaces reflect different logical forms as opposed to the *one* logical form of the *Tractatus*. I agree with Conant that this "open-ended, infinitely extendable conception of a family of possible forms of grammar comes to be seen to require a correlatively open-ended, infinitely extendable conception of a family of possible forms of philosophical method" (2011, 641). But I do not see this methodical pluralism only develop in the late Wittgenstein. If he had said in his 1930 lecture that there must be many methods to account for all voices, it would not have contradicted his general thinking at that time. A difference of form and grammar is not a difference between things of the same kind which can be treated with the same method – this is the gist of the manuscripts from 1929–32 as I will show in chapter 4.

The specific narrative Conant presents is solely based on Lee's lecture note about a method whose intricacies are nowhere expounded. The obscure statement that now there can be "skilful" philosophers, is pushed too far when he argues that for the middle Wittgenstein "no fundamental form of originality will any longer be required on the part of the philosophical practitioner in order for him to be able to make genuine progress with philosophical problems" because "a method has been found" which philosophers can, as it were, mechanically follow (Conant 2011, 640). On the contrary, the *invention* of new forms, of new spaces, is an important characteristic of Wittgenstein's thinking from the early 1930s, especially in his philosophy of mathematics as I will show. Far from being mechanical puzzle-solving, the development of grammatical forms, which are often conceived as spaces in the middle period, involves a considerable element of creativity.

9. Nor is it clear what the many methods of the late Wittgenstein are even though here there are plausible suggestions. Oskari Kuusela, for example, provides a list of Wittgenstein's methods including the method to use rule-governed calculi and language-games as objects of comparison and the method to pursue philosophy like "natural history" – which obviously captures Engelmann's "anthropological" ideas. Kuusela also suggests that there is a sense in which Wittgenstein's later method is *one*, but can take various forms depending on the philosophical problem at hand (cf. Kuusela 2008, 269–270).

Neither Wittgenstein's "space" nor "grammar" or "calculus", which Engelmann seems to regards as universal concepts, are means of reductionism. Rather, these concepts are the expression of the post-Tractarian pluralism with regards to both forms and methods. Both Engelmann and Conant seem to be eager to present the middle Wittgenstein as somewhat mechanical and obsessed with fixed rules whereas the late Wittgenstein is, as it were, full of life and humanity. Even though spaces and calculi may appear dry in contrast to the more animate "games" and "forms of life" that inhabit the *Investigations*, the idea to compare language to such systems had a very similar function as the later, more liberal, concepts; vagueness of formal systems and the diversity of according methods were already discussed in the early 30s. Against Engelmann, I argue that there is indeed a plurality of grammatical spaces in the middle period, and against Conant, that these spaces correspond already to a plurality of methods.

By focusing on space, I emphasise this continuity. Since it is such a flexible concept in the middle period, which is used in a number of different philosophical fields, my claim is that it is a central characteristic of Wittgenstein's thinking and that it is not dismissed in the late works as Engelmann and Conant suggest. The set of problems that this metaphor is supposed to capture motivates the introduction of a "logical space" in the *Tractatus*, it leads to modifications in the middle period, and it survives in the late period although it is then no longer conceived of in terms of spaces. Hence, the spatial perspective allows me to give a new account of the similarities and dissimilarities between the various stages of Wittgenstein's philosophy without marginalising the concepts of the middle period or neglecting their later transformation.

Despite his earlier view, which regarded spaces as a transitory phenomenon in Wittgenstein's development, Schulte acknowledges, in a more recent paper, a connection between the logical space of the *Tractatus* and the later use of space concepts that are connected with "grammar" (cf. 2017, 319).[10] This connection has also been noticed by other scholars (e.g. Glock 1996, 223; Hyder 2002, ch. 7; Ometiţă 2018), but the similarity has not been spelled out yet and it has not been made clear what motivates the usage of the common term "space" – which is itself, just like its sister concept "time", a highly controversial term in philosophy. Drawing on its history in Kantian and Neo-Kantian philosophy, I shall argue that Wittgenstein's logical space in the *Tractatus* is more than a metaphor and more than a local phenomenon; thanks to its technical functions, it is suited to take up a central role in the revision of his earlier work in the middle period; since functions of spaces remain vital in the late writings, I use them as a means to illustrate the continuity of his philosophy.

10. Schulte's example is even from the early 1940s which contradicts his earlier claim that spaces disappear after 1933.

Wittgenstein himself seems to have seen a kind of unity in his philosophical life. Naturally, such an observation can only be made retrospectively and it is indeed in the writings from his last years where we find such comments on his own work (the motto by Nestroy was added after 1945, too). In 1951, shortly before his death, he wrote the remark whose original version I used as an epigraph for this introduction: "I believe a philosopher, someone who can think for himself, could be interested in reading my notes. For even though I rarely hit the bull's eye, he would recognise at which targets I was ceaselessly aiming" (MS175, 64v—65r). This suggests a more or less stable set of problems Wittgenstein was struggling with, not only in his last writings, but "ceaselessly" (*unablässig*). In 1946, he wrote the remarkable sentence that inspired Steve Reich to his musical composition *Proverb*: "How small a thought it takes to fill a whole life!" (MS131, 180). In its context it is plausible to interpret this remark as referring to Wittgenstein's own philosophical life; the "small thought" is a bit obscure (the preceding sentence is "The forces that determine *how* something is represented are as great as those that insist on the truth of the representation"), but by the end of this thesis I shall be in a position to propose an interpretation of that thought which connects it to space and takes it as a nodal point for Wittgenstein's whole philosophy where many interests and problems intersect. I shall achieve this position by looking at the role of space in writings from all stages of his work.

1.2 Structure of the Thesis

In the second chapter of the thesis, I shall look at the roots of Wittgenstein's space concept. I shall investigate the relation between space, geometry, and *a priori* in general which has been central to the work of many 19th century mathematicians and scientists and to the logical positivists in the 20th century. Before approaching the peculiar notion of logical space, it is helpful to consider some precursor ideas of it. Even though they may not have influenced Wittgenstein directly (some certainly have), they still set the stage for the modal ontology of the *Tractatus*.

I start with a brief sketch of Kant's philosophy of space in the *Critique of Pure Reason*. Space is here seen as a "form of intuition" which is not itself an object, but a condition of the possibility of perception. With a Wittgensteinian phrase one could say, this form is "the possibility of a structure". Moreover, space and time are Kant's prime examples of *a priori* knowledge which resembles in some ways the Tractarian account of logic, although its insights are supposed to have a different kind of *truth* than the apodictic certainties Kant held to be *a priori*. Another parallel is Kant's belief that Euclidean space is *the* form of intuition just like Wittgenstein believed logical space reflected *the* form of language and reality.

In the middle of the 19th century, non-Euclidean geometries were developed in which at least one axiom (or postulate) turned out to be not necessarily true (the parallel postulate). Helmholtz, drawing on Riemann, develops an account of space-like manifolds to account for sensory experience. Generally, geometry becomes more abstract: it is no longer the study of a three-dimensional Euclidean container in which supposedly everything physical takes place, but an unspecified manifold of n coordinates and with variable metrics. The formalisation of geometry led to a deflation of the concept of *a priori* and it enabled Wittgenstein to fruitfully employ spatial imagery in his first philosophical book. Before he used geometrical methods in his *Tractatus*, physicists such as Hertz and Boltzmann had used ideas by Kant and Helmholtz to provide instructive representations of states of physical systems. Again the idea of spatial locations as possibilities is central. The application of this method to physics can be seen as a result of the increasing abstraction of geometry in the middle of the century: if "space" is no longer the actual three-dimensional structure we live in, but an abstract manifold of possibilities, then it can be used to represent other possibilities as well: in Hertz's state space particles can be located within a physical system and each possible state of the system can be described as one momentary totality of these particle-coordinates; in Boltzmann's phase space statistical distributions can be modelled in space-like $6n$-dimensional manifolds. Both physicists also developed concepts of "picture" (*Bild*) for certain scientific frameworks that may have influenced Wittgenstein; however, I argue that the influence of their picture theories is rather limited, while their, roughly speaking, Kantian ideas about space are more relevant for an understanding of both picture and space.

In chapter 3, I investigate how Wittgenstein builds on this development in his *Tractatus Logico-Philosophicus*. The aspects I want to highlight in my interpretation are the focus on possibilities, the role of negation and logical operations in general, and the claim for unity and generality. *First*, I shall describe the Tractarian "ontology" of objects, atomic facts, and facts and their interdependences; the isomorphism between language and world in virtue of the shared logical form which is to be understood in the Kantian sense as the possibility of a structure: the possible structures of words in propositions correspond to the possible structure of objects in facts (the picture theory). *Second*, I look at one important function of logical space besides representing possibilities, namely to illustrate the function of logical constants without them being represented as "objects" (Wittgenstein's "fundamental thought", TLP 4.0312). Negation can be conceived as mutually exclusive (and exhaustive) regions of a space; conjunction and implication are relations between states in a state-space similar to Hertz's. *Third*, I investigate the claim for unity in the *Tractatus* and the idea that the described language mirrors the necessary structure of the world. Again, Kantian ideas are involved in these arguments, even in the terminology (e.g. TLP 6.13: "logic is transcendental"). The

claim to have found the "general form of the proposition" (TLP, 4.5; 6) suggests that there is only *one* kind of proposition, that it is hence possible to represent its general form, and that this representation could be achieved by means of a universal structure such as Wittgenstein's logical space.

In the fourth chapter, I look at the use of space concepts in the middle period. While keeping many of its functions, logical space loses its uniformity and disintegrates into a plurality of grammatical spaces with sometimes incompatible context-dependent rules. These grammars, which are repeatedly compared to geometries, still serve as frameworks of possibilities and also inherit the crucial distinction between geometry, which is strictly speaking senseless, and meaningful propositions within a space. What is important for Wittgenstein at this stage is to keep these geometries apart and to show where confusion of spaces leads to philosophical problems. He tackles a number of related fields in these years while always referring back to the imagery of space and its basic distinctions. In mathematics, he distinguishes number spaces and insists on the creativity that is involved in developing new calculi, in "inventing" new spaces, rather than "discovering" places in an already given space. Wittgenstein's treatment of colours in the middle period is said to be a milestone insofar as it showed the limitations of the Tractarian account and suggested the need for a kind of phenomenological space of colours. Phenomenology in a wider sense is opposed to physicalistic conceptions of language – generally, the space of physics is contrasted with visual space as an example that involves vagueness and is not a "measuring space" and hence, *pace* Conant, requires different philosophical methods. Finally, Wittgenstein employs space metaphors when discussing intentionality, the mind's relation to its objects: against causal theories of meaning and theories about mental representation of the world, he highlights the limitations of language and its dependence on a systematic grammatical space in which alone thought and reality can meet.

Having carved out the functions of spaces in the middle period, I can in the fifth chapter recognise other concepts as belonging to the same set of problems. Formal systems that are comparable to spaces are conceived in terms of a variety of typical Wittgensteinian concepts which supports my claim that space as a problem, not necessarily as a word, plays an important role in his whole philosophy. The first group of these concepts can be described as *complementing* the functions of space. A "form of representation" clearly has similar functions insofar as it determines the way we look at things. The discussion of notations in the middle period can be seen as a pluralist modification of the notation of the *Tractatus* which was supposed to be *the* correct one. Even theories, which tend to be dismissed in Wittgenstein scholarship, can have the grammatical function of what is elsewhere called a space. A second group of concepts *replaces* space concepts as can be proven empirically –

by looking at their occurrences in correlation with those of space – as well as conceptually by showing how they take up functions of space. These are especially calculi and games which gradually become more important in the form of language-games, a central tool of the later Wittgenstein. The third group are larger frameworks which resemble the universal logical space, but are now presented in a relativistic framework. Forms of life are indeed larger patterns of human organisation, but none of them can claim universal validity or truth. Like space in its deflated sense, they rather determine a way of looking at things. A similar role is attributed to concepts like style and world-picture.

In the sixth chapter, I discuss limitations of space concepts which partly motivate their substitution by other concepts. Unsatisfied with Engelmann's account of the anthropological view replacing grammar and with Conant's emphasis on the number of methods, I propose my own interpretation of the relation between the middle and the late Wittgenstein. This interpretation is two-fold. Its first aspect is a shift from static spaces to more dynamic frameworks; the second is the acknowledgement of the role of empirical, sometimes anthropological, elements in the late writings. Thanks to the focus on space, however, I can still see the continuity in these developments without regarding the new ideas as replacements of the older ones. Instead, I argue, not unlike the polyphonic readings by Pichler and Stern, that in the later Wittgenstein a dialectic is at work that presents arguments between different positions none of which is given preference in a clear-cut way.

The thesis is primarily a contribution to Wittgenstein studies, especially to the debate on continuity in his work, but beyond that I hope to open up perspectives for an application of the gained insights. Since Wittgenstein is widely considered one of the most important philosophers of our time, a clearer understanding of his philosophy should have ramifications beyond the merely exegetical debate. Directions in which such applications could lead are indicated in the conclusion.

1.3 Some Remarks on the *Nachlass*

Since my approach aims to provide an interpretation of Wittgenstein's philosophy as a whole and to trace its continuities and developments, it demands a closer look at the structure of the writings themselves. The *Tractatus* has a unique position in this oeuvre as the only completed and published work by Wittgenstein that we can take as his last word (at least at the time of its publication). Nevertheless, the war-time manuscripts and letters in which he explains parts of the book can be a great help to illuminate the very brief, dense and sometimes puzzling remarks of the *Tractatus*.

After 1929, the situation becomes more complicated: there is a series of manuscript volumes which, in my opinion, best reflect the continuous thinking process, especially between 1929 and 1932. Other writings of this time, which have received much attention in the literature as they present some of Wittgenstein's thoughts in a more accessible – sometimes deceivingly straightforward – way, shall be less important in my thesis. Lecture notes by students and conversations with the Vienna Circle mostly reflect arguments that occur more comprehensively in the manuscripts while blurring or concealing their clearly preliminary status in Wittgenstein's development; besides, they are "second hand" sources which do not have the authority of an original document. Texts by Wittgenstein himself from that time, such as the paper *Some Remarks on Logical Form*, the typescripts on which the publication of the *Philosophical Remarks* is based (TSS208, 209) and which were later used for the compilation of further typescripts, seem likewise to be side products of the flow of arguments in the manuscripts with only small, and only rarely significant, deviations or differences in emphasis (the order of TS209, for example, can be significant). These typescripts were eventually joined by Wittgenstein to one big collection, the *Big Typescript* which, however, does not contain substantially new material compared to the manuscripts. It can be instructive through the way in which it is structured. As the only text by Wittgenstein it features a table of contents and is structured in chapters and subchapters. The order of these chapters and the assignment of an individual remark under a certain heading can give that remark a new gist which sometimes makes it appear more clearly as a step in an argument (and sometimes the opposite: sometimes remarks function well in their original context in the flow of the manuscripts and stand somewhat erratically in a chapter between remarks that seem unrelated). The manuscript volumes and the *Big Typecript* have been critically edited in the *Vienna Edition*. For referencing, I shall use von Wright's codes for the manuscripts and typescripts (cf. G. v. Wright 1969) because the *Vienna Edition* does not cover the later manuscripts to which I will draw frequent connections. When translating from the manuscripts, I shall indicate cancellations and variants only where I regard them important for the argument made. Generally, my translation is geared to the normalised transcription from the *Bergen Nachlass Edition*.[11]

What is interesting in the context of the *Big Typescript*, is how Wittgenstein used it after 1933 when he started new attempts to summarise his ideas in the form of a book, e.g. in the network of manuscripts that has been published as *Philosophical Grammar* or MS116 which starts as a revision of the *Big Typescript* and gradually becomes more and more like the *Investigations*. In the middle of the 1930s, there is also the *Blue Book* which is more reliable than the lecture notes as it has been dictated by Wittgenstein himself; it contains

11. It is available online at wittgensteinonline.no and linked to the facsimiles at wittgensteinsource.org.

in consecutive prose some ideas that are spread across various remarks in the manuscripts. The *Brown Book*, dictated one year later, can be seen as the start of the project *Philosophical Investigations*, but is still very much based on the material from the first ten manuscript volumes 1929–1932. A period in Norway in 1936/37 seems to mark the start of the actual corpus of the *Investigations*: again, the writings from this time are to a large extent concerned with ideas from earlier work, from the *Tractatus* and the *Big Typescript* which he seemed to have brought with him to Norway, but the tenor seems to have taken a new direction. From this time, notebooks are particularly useful, pre-stages of the manuscript volumes which exhibit Wittgenstein's ideas, as it were, in *statu nascendi*. The focus on the foundations of mathematics, which follows this phase well into the 1940s, is treated here not as a break, but as a continuation from the lines of thought that have started in the 30s. They can be put in terms of the "spatial" themes that I will focus on: possibility and surprise, the role of *a priori* knowledge, the nature of (grammatical) rules, games and practice.

The *Philosophical Investigations* are rightly seen as Wittgenstein's second masterpiece although they have not been published by himself. I will use the latest German-English edition by Hacker and Schulte and Schulte's German edition which includes all pre-stages and revisions and thereby does justice to the uncompleted status of the book. The last writings, manuscripts which led to the publications of *Remarks on Colour* and *On Certainty*, but also the so-called part II of the *Investigations*, are not a "third Wittgenstein", but reveal quite continuous accounts of language-game, *a priori* knowledge etc. which are related with space as I understand it.

Chapter 2

Spaces in Kantian and Neo-Kantian Philosophy

Kants Größe beruht auf der Konzeption des Begriffs einer "Form a priori", aber nicht auf der Anwendung, die er ihm gab.

Oswald Spengler
Der Untergang des Abendlandes

This chapter has two aims which are intimately intertwined with one another. The first follows naturally from my ambition to understand Wittgenstein's use of spatial metaphors and concepts: to trace the origins of this imagery. As merely pointing to the usual suspects does not seem to be satisfactory – recent scholarship rather highlights the differences between Wittgenstein and Hertz or Boltzmann –, I go a bit further back in the history of philosophy, notably to Kant's notion of space and to the related concepts of *a priori* and *a posteriori*, necessity and possibility. Of course, I am not the first to propose connections between Kant and Wittgenstein, but my focus on the development of Kantian space-concepts sheds some new light on this kinship and will be a constant point of reference in the chapters to come.

This touches the second aim of this chapter: to clarify or at least sketch aspects of the sense in which "space" shall be used in the rest of the thesis. The use of spatial expressions in the *Tractatus* seems to be metaphorical; indeed an important instance, the "space of possible atomic facts" (TLP 1922, 2.0131)[12] is introduced with a typical marker of imagery: "*gleichsam*", translated in both English editions with "as it were". So one objection against

12. In the following, the quotes from the *Tractatus*, unless otherwise specified, are from the 1922 edition and translation which has been authorised by Wittgenstein. Problems of this translation shall be addressed in chapter 3.

a special focus on spatial expressions might be that they simply have a local illustrative purpose like other metaphors Wittgenstein employs. However, there are two reasons that make logical or grammatical "space" special. First, the sheer number and the argumentative role of its occurrences distinguish it from merely local tropes or images. This claim shall be substantiated in chapters 3 and 4 whose topic is the usage of "space" in the *Tractatus* and in the middle period. Second, while other Wittgensteinian images, "ladder", "spade", "fly-bottle", have themselves histories of controversial interpretations and debates, they are more or less straightforward metaphors which compare an abstract idea with a more concrete physical object and receive their illustrative power from this concreteness.[13] One might argue that this "direction" is still at play in the case of space since what is illustrated with this image is the most abstract and general idea of all, namely the possibility of "the world" conceived as a totality of logical facts. But there remains the problem that "space" is itself a highly abstract concept and that its *tertium comparationis* is not immediately obvious. To be sure, spatial metaphors are not a Wittgensteinian peculiarity: "up" and "down", "far" and "close" are ubiquitous in conceptual discourse as well as in everyday language. But when "space" and "geometry" themselves are used as analogies something different is happening. They are themselves philosophical terms with an intricate history and controversial interpretations, with varying functions in different philosophical approaches. Therefore the way the analogy works depends on the respective interpretation of space and geometry on which the metaphor is based. To understand the sense in which these unusual metaphors are employed by Wittgenstein, it is helpful to explore how "space" and "geometry" have been used by writers with whose ideas he was, directly or indirectly, familiar. The possible influences I consider in this chapter have already been linked to Wittgenstein in the literature in one way or another. Similarities between Wittgenstein and Kant have been the topic of numerous papers and discussions; Hertz and Boltzmann have repeatedly been associated with the *Tractatus* philosophy. My focus on spaces draws out another narrative which includes these thinkers, but also considers the development of 19th century geometry in general whose intersections with Wittgenstein's philosophy have received less attention in the literature.[14]

What I want to argue is that "space" – like "picture"[15] – is not merely a metaphor, but a technical, a formal concept at the heart of Wittgenstein's philosophy which can be better understood against the background of Kantian philosophy and especially its later development

13. The "direction" of conceptual metaphors typically goes from concrete to abstract (cf. Kövecses 2010, 7).

14. An exception is David Hyder's profound account of structural similarities between Neo-Kantian philosophy and the *Tractatus*. His book *The Mechanics of Meaning: Propositional Content and the Logical Space of Wittgenstein's Tractatus* (2002) is an important source for this chapter.

15. " I have inherited this concept of a picture from two sides: first from a drawn picture, second from the picture of a mathematician, which already is a general concept. For a mathematician talks of picturing in cases where a painter would no longer use this expression" (WVC, 185).

and modifications. In the first section, I shall focus on Kant who provides the terminological and systematic background of both the Neo-Kantian and the Wittgensteinian ideas. Unlike most comparisons to Kant (cf. Hacker 2012a; A. Moore 2013; Sullivan 2013), I do not focus on the *Transcendental Deduction*, not on solipsism and the transcendental unity of apperception, but on the first parts of *The Critique of Pure Reason,* on the introduction and on the *Transcendental Aesthetic* where space and time are discussed as pure forms of intuition. There is an important kinship between Kant and Wittgenstein's uses of space which I want to highlight and which consists in their respective treatments of necessity and possibility and of possibility and actuality.[16] But, to anticipate an obvious objection, logical space in the *Tractatus* does not deal with Kantian intuition; and of course, Wittgenstein does at no point embrace the idea of any truth that is "synthetic a priori". On the contrary, one of his core claims is that logical propositions, which were held to be the most general truths about the world by his predecessors (e.g. Russell), are "true" in a different sense: for they famously "say nothing" and are true only in virtue of their form like Kant's analytical propositions. While Kant would have agreed that logical propositions are empty, mathematical propositions, synthetic *a priori* knowledge for Kant, are taken to be pseudo-propositions by Wittgenstein (TLP, 6.2); and neither is "pure science" considered an *a priori* truth: "outside logic all is accident" (TLP, 6.3).

These insights resemble the development of geometry after Kant. For Kant the axioms of geometry still said "something" for they were synthetic *a priori* truths: they required intuition, if only "pure" intuition. The development of geometrical conventionalism in the course of the crumbling universality of geometry, shall be dealt with in the second section of this chapter. Kant's space, which is describable with Euclidean geometry, was questioned and attacked throughout the 19th century by mathematicians, physicists and philosophers. Some completely rejected the Kantian ideas, some tried to modify them in light of new insights in mathematics and physics – a questionable enterprise since one aspect of Kant's classification of geometry as *a priori* was that it is immune to empirical results and that its mathematical insights are of timeless necessity. However, the idea was precisely to shift this necessity from the specific structure of Euclidean space, which Kant held to be the form of the outer sense,[17] to a more abstract level of necessity. The homogeneous three-dimensional space without curvature was no longer an apodictic certainty, but there is a higher – perhaps trivial

16. The *Deduction* may be relevant for these issues, too, but to keep the focus on space it makes sense to stick with the *Aesthetic* (after all, it is itself a kind of transcendental deduction of the concepts of space and time, cf. A85–90, B118–122). And it was geometry which became the prime example for *a priori* knowledge and its lost universality.

17. Kant scholars might object that Kant's philosophy of space changed a lot during his lifetime and not at all stages did he claim the apodicticity of Euclidean space (cf. Caygill 1995, 367–373). However, this became the most influential of his positions and the one that was associated with the Kantian *a priori* in the 19th century.

– necessity that any geometrical system, Euclidean or non-Euclidean, must be a manifold of coordinated dimensions.[18] The axioms, the number of dimensions, and the curvature can vary from system to system, but once a certain geometry is in place its axioms can be called necessary; however, they are necessary only relative to the preceding *choice*. Such an understanding of geometry is first of all a mathematical notion and is less bound to physical space which Kant was aiming to make intelligible. This abstracted notion of geometry lends itself already more easily to metaphorical and technical applications and Helmholtz's idea of space-like manifolds of perception is one of them.

While we can only speculate about Wittgenstein's direct acquaintance with this development, his acknowledged influences, Hertz and Boltzmann, draw heavily on it and use abstract and multi-dimensional "spaces" in order to model physical systems. This shall be the topic of the third section. Wittgenstein's familiarity with this abstract understanding of "space" is likely to stem from these two thinkers; however, the structural similarities with this tradition exist irrespective of any direct biographical influence. Besides the historical account of philosophies of space and geometry from Kant to Wittgenstein, the aim of this chapter is to explain how space could take up the metaphorical and yet technical functions it features in the *Tractatus* and in the writings of the middle period.

2.1 Kant: Space and Form as Possibility

Wittgenstein's relation to Kant is controversial. Parallels between their philosophical projects have been recognised from the beginnings of Wittgenstein scholarship (cf. Stenius 1960) and have attracted interpreters ever since. There certainly are significant similarities between the two thinkers. Peter Hacker, who suggested very strong ties between them in the first edition of his book *Insight and Illusion* (1972), describes these affinities more modestly in the second edition:

> Both philosophers shared a conception of philosophy as concerned with the bounds of sense, even though their conception of what determines the latter differed. Both thought that many propositions of traditional metaphysics violate the bounds of sense, misuse concepts, and hence make nonsensical claims. More than any other philosophers, Kant and Wittgenstein were concerned with the nature of philosophy itself and sought to curb its metaphysical pretensions by

18. Among others, Russell, in an early paper on the *a priori* in geometry, distinguished necessary axioms of any geometry and Euclid's axioms which could only be derived from experience (cf. 1895). However, he did not go as far as the thinkers I am going to consider. For example, he held the Axiom of Free Mobility as *a priori* necessary for any geometry, while Helmholtz thought of it as an empirical matter and Riemann even considered geometries without this axiom.

> clarifying its status and circumscribing what one may rationally hope for in philosophical investigation. Both saw philosophical and metaphysical *illusions of reason* as at least a large part of the subject, and the eradication of such illusions as a major goal of their work. And they shared a highly critical attitude towards traditional empiricism and rationalism alike. (Hacker 1986, 207)[19]

Comparisons between Kant and Wittgenstein have mainly focused on their philosophical methods and their reflections on the role of philosophy. The "critical" project of Kant to draw a limit to pure reason seems to echo in the Tractarian project "to draw a limit to thinking – or rather, to the expression of thoughts" (TLP, preface). There is also a striking Kantian ring in the sub-remarks to proposition 6 of the *Tractatus*, where Wittgenstein describes logic as "transcendental" (TLP, 6.13), then comments on the Kantian examples of *a priori* knowledge, mathematics (in the 6.2s) and science (in the 6.3s), before turning to ethics and aesthetics which are, again, "transcendental" (TLP, 6.421).[20] An elaborate interpretation of "Wittgenstein's Kant" cannot be constructed out of these remarks, but a certain affinity cannot be denied. Moreover, the general philosophical attitude, the drive to "curb metaphysical pretensions" and to "eradicate illusions" seems to be a constant throughout Wittgenstein's thinking and is not restricted to the *Tractatus*. The *Investigations* are likewise concerned with the limits of sense, although here it is investigated more closely what kinds of limits we are talking about in philosophy (cf. PI, §§499, 500).

Despite these similarities, Kant is not mentioned in the famous list of influences that Wittgenstein had created himself in 1931 (cf. MS154, 16r);[21] and direct references to Kant are rare in the over 20.000 pages of his *Nachlass*. According to Ray Monk's biography, he studied the *Critique of Pure Reason* together with Ludwig Hänsel in a prisoners of war camp in Italy in 1919 (cf. Monk 1990, 158), but there are few traces of this reading. In any case,

19. More recently, Hacker has elaborated on these affinities, focusing especially on the question whether Wittgenstein's arguments can be called *transcendental*. In a strict sense, they are not, Hacker concludes, but he also repeats the similarities to Kant which he had already listed in 1986 (cf. Hacker 2012a).

20. Peter Sullivan notes the Kantian background of these topics (cf. Sullivan 1996, 198). He takes these remarks to be a step-by-step refutation of Kant's synthetic *a priori* and holds that Wittgenstein here engages with Transcendental Idealism, but in the end rejects it. Peter Hacker, in the above-mentioned paper (Hacker 2012a), warns against calling the Tractarian argumentation "transcendental": there may be structural similarities, but it does not match all the implications this technical term has in Kant. While I can agree with these "anti-Kantian" readings with regards to a direct connection of doctrines ("Wittgenstein: a transcendental idealist"), I still argue for a structural similarity. So much is clear: Wittgenstein rejected any non-logical necessity (TLP, 6.37) and therefore also any form of synthetic *a priori* knowledge; the *Tractatus* rejects any "a priori order of things" (TLP, 5.634). There is no necessary form of intuition as in Kant, but there is a necessary form of the proposition. The metaphorical transfer I am interested in is from experience to logic, which is done by means of the concept of "space". While the Kantian space was a framework of empirical possibilities, Wittgenstein's space is a framework of logical possibilities.

21. Boltzmann and Hertz, the first two influences on this list, are considered in section 2.3.

Wittgenstein must have been familiar with Kantian thinking before 1919 through the writings of Schopenhauer,[22] probably through Russell's treatment of Kantian ideas, and through his studies of so-called Neo-Kantian philosophers and physicists who shall be considered in the later sections of this chapter.

Leaving revisions aside, there are only four direct references to Kant in the *Nachlass* :

- In 1914, Wittgenstein writes that the theory of tautologies might throw light on Kant's question "how is pure mathematics possible" (MS101, 51r).
- Later in the war (MS104), he considers Kant's problem about the right and the left hand which cannot be made to coincide (Prolegomena, §13).[23] This remark becomes part of the final version of the *Tractatus* (TLP, 6.36111).
- In the writings from 1929–1932, he repeatedly compares his views on mathematics to Kant's claim that mathematical propositions are synthetic *a priori* (MS107, 183; MS114, 10r; transferred to TSS 208, 209, 211, 212, 213).[24]
- In 1931, he refers somewhat vaguely to the "Kantian solution to the problem of philosophy" which he holds to be related to his insights about the limits of language (MS110, 61).

For my purposes, it is helpful to narrow down the focus on Kant's terminology and categorisation of judgements in general and on his treatment of space in particular. Questions of ethics and religion, which were of highest importance for Kant as well as for Wittgenstein, shall not be touched at this point. This is arguably in line with their respective philosophical programmes which are at pains to delimit those higher spheres mainly negatively (despite some dissimilarities in *how* they delimit them).[25] I shall also not deal with the general methodological similarities that have received so much attention in the literature. For

22. Schopenhauer is the third thinker mentioned in the list of influences and he commented on Kantian philosophy extensively, especially in the appendix to *The World as Will and Presentation.* Wittgenstein's admiration for Schopenhauer, at least in his youth, is well documented and researched, e.g. Lange 1996; Jacquette 2017. It is possible that the early Wittgenstein knew Kant largely through Schopenhauer. The order of the 6.n propositions, related by Sullivan to Kantian topics, can also be seen as a response to the *Fourfold Root of the Principle of Sufficient Reason* where Schopenhauer distinguishes between reasons in logic, mathematics, science (causality) and human actions (motivation) which corresponds to the topics from 6.1 to 6.4.

23. Wittgenstein might have this not from a direct reading of Kant, but from his conversations with Russell who dealt with Kant's treatment of the asymmetry of spatial relations for example in chapter XXVII of the *Principles of Mathematics* (1903).

24. I will get back to this comparison when I discuss mathematics in Wittgenstein's middle period (section 4.2).

25. The dispute between Sullivan and Adrian Moore centres on this problem: Wittgenstein conceives of the bounds of sense as "limits", that is, as something non-contrastive where the other side is not known. Moore points out that towards the end of the *Tractatus* this limit is recast as a "limitation", a contrastive boundary

now the focus shall be on technical questions concerning the synthetic *a priori* and space. Wittgenstein's explicit references to Kant, quoted above, all belong to this field as we will see.

The "spatial" perspective reveals some lesser-known and less obvious similarities between the two philosophers: Kant was maybe not the first, but a significant champion of the view that space (like time) can be considered a *framework of possibilities* which thereby provides a connection between mind and world. The *a priori* restrictions of the mind determine the possibilities of experience which can then be confirmed or confuted by actual, empirical input. Let me first consider the technical terms Kant introduced to categorise judgement. These are designed to account for the quarrels between rationalists and empiricists he was responding to: is all our knowledge created by the mind, or do we altogether depend on experience?

Kant's categorisation of judgements is meant to capture what is plausible in both views. Some knowledge can apparently only be acquired after empirical reality is perceived. Whether a lamp is on the table, is a question of experience and a true proposition stating this fact is a form of *a posteriori* knowledge. However, there is also knowledge that is independent from experience. Judgements that belong to this type are called *a priori*. These fall into two further types. We can tell that a collie is a dog without looking at empirical facts first. The truth of this statement about collies is implied by the concept itself; knowledge of its truth is a form of *analytic* knowledge which is always *a priori*. However, not all *a priori* knowledge is of this kind. Kant holds that mathematical knowledge is different. A mathematical term like $5+7$ does not entail its result 12 in the same way the concepts of collies and greyhounds entail the concept of dogs. The result cannot be found by analysing the summands, but requires a form of intuition (*Anschauung*). This intuition must, however, not be empirical as we clearly do not depend on experience when conducting a calculation. The truth of a mathematical judgements is, according to Kant, *synthetic a priori* (cf. CPR, B4). Hence he has an example of synthetic *a priori* truth, the kind of truth he is interested in to achieve his objective to give metaphysics a scientific footing. Since synthetic *a priori* knowledge is possible, as is proven by the possibility of mathematics, Kant's central question in the first *Critique* as well as in *Prolegomena* is: *how* is synthetic *a priori* knowledge possible?[26]

Qua *a priori*, a mathematical judgement cannot stem from empirical intuition which depends on experience; and by being synthetic, its truth cannot lie in the parts of the

between facts and values. Since this pattern resembles the Kantian project to delimit reason in order to make room for faith, he calls Wittgenstein a Transcendental Idealist. Sullivan, in turn, disagrees because ethics is categorically different from facts and not just on the other side of a shared border. The limits of facts remain "limits" in the sense sketched above (cf. A. Moore 2013; Sullivan 2013).

26. Wittgenstein's mention of "the Kantian solution of the problem of philosophy" (MS110, 61) might refer to a version of this question as he was apparently familiar with the basic ideas of Kantian epistemology.

judgement themselves. It has to derive from *pure* intuition. In the *Transcendental Aesthetic*, as well as in *Prolegomena* (§10), Kant argues that *time* and *space* provide this pure intuition by being its *pure forms*. We cannot perceive time or space themselves, but everything we do perceive we perceive through these forms. Intuition stripped of any empirical content is pure intuition, and geometry studies the pure and formal properties of space, while arithmetic studies the pure and formal properties of time (numbers as the basis of succession).[27] By being pure forms of intuition, space and time are the *conditions of possibility* of empirical intuition. This form of knowledge, which does not deal with objects, but with the *a priori* possibility of a kind of knowledge, is called *transcendental* (CPR, B25/A11). Later in the *Critique* Kant argues that in "pure science" the mind constrains what is possible, too. Its *categories* determine what can be known about the physical world, they form a transcendental scheme of possibilities which can be "filled" with actual knowledge by empirical input. One of the laws that can be derived from transcendental insights of this kind is the Law of Causality: as *a priori*, it is not itself empirical, but constitutes empirical knowledge (cf. Caygill 1995, 108).

These Kantian ideas clearly resonate with Wittgenstein's use of "transcendental" in the *Tractatus*.[28] A transcendental logic is one that does not deal with objects nor with specific empirical propositions, but determines what can count as *possible* propositions in the first place. The analogy of logical space takes up and modifies the Kantian idea of space as a form of intuition, that is, as the structure which determines the possibilities of empirical intuition. Tractarian *form* is repeatedly characterised as the "possibility of a structure" (TLP, 2.033; 2.15). Obviously, logical structure is not the same as spatial structure, but the analogy is that relational possibilities – be they about logical or spatial relations – are in both cases "constrained" by form. Here we encounter what Wittgenstein calls the limits of language: the limit of possibilities in this sense has the peculiarity that we know only one side of this limit: that we cannot think what is impossible.[29] While all propositions proper *within* logical space are *possible*, the logical form itself is *necessary* and cannot be thought or said, it can only be shown. Logical propositions, that is tautologies and contradictions, can therefore be viewed as "part of the symbolism" (TLP, 4.4611),

27. At least in the *Prolegomena* this kind of parallelism seems to be implied (§10). For an account that highlights the differences between geometry and arithmetic in Kant, see Potter 2000, ch.1.

28. It is unfortunate that the German words "transzendental" and "transzendent" are both translated as "transcendental". One might take the Tractarian "transcendental" in the second sense, that is, simply as transcending the bounds of a certain sphere – which is of course correct: logic, ethics, aesthetics do indeed transcend these bounds, but not because they are "outside" of them, but rather because they constitute them. They are conditions of possibilities. The Tractarian world is a world of possibilities ("what is the case" is contingent); investigating its conditions, logic and philosophy, is therefore a transcendental enterprise in the first sense although it deviates from Kant's use of the word in other aspects (cf. Hacker 2012a).

29. See the dispute about limits and limitations between Sullivan and Moore mentioned above.

although they say "nothing" (TLP, 5.43). They, as it were, constitute logical possibilities and can in this sense be compared to the Kantian forms of intuition: space and time, which constitute empirical possibilities.[30] To be sure, Wittgenstein's problem is not epistemological and he does not talk about intuition in the *Tractatus*,[31] but the relation between form and facts, between possibility and reality is strikingly similar. This connection also provides an explanation for Wittgenstein's early remark, that the theory of tautologies could illuminate the Kantian question of how mathematics is possible: mathematical propositions, like logical propositions, do not say anything about the world, do not "express a thought" (TLP, 6.21), but constitute a framework of equations which, like logical tautologies, "show the logic of the world" (TLP, 6.22).

The repeated references to *the* world, *the* language, general form and logical space which are used in a universal singular, reveal a certain claim for universality in the *Tractatus* which is likewise shared with Kant's account of space (and his account of *a priori* knowledge in general). *A priori* knowledge about space and time is held by Kant to be necessarily and universally true (cf. Prolegomena, §12) – a transcendental certainty which derives from the conditions of possibility of experience.[32] It is above all this universal validity of Euclidean geometry which has been associated with Kant by geometers in the 19th century; the criticisms thereof shall be regarded in the next subsection. Wittgenstein's logical propositions are necessarily and timelessly true in a similar way and, like Kant's *a priori*, they are not refutable by empirical knowledge. A certain anti-Kantian twist can be seen in Wittgenstein's view that empirical laws (such as Newton's mechanics) as discussed in the 6.3s are not necessary. Another major difference between the philosophers, which has become evident in the above, consists in their respective treatment of synthetic *a priori* knowledge: for Kant necessary propositions of this kind were necessary *truths*, while Wittgenstein would in large part deny their truth-aptness or at least highlight the peculiar way in which they are "true" which is very different from empirical truths (cf. Hacker 1986, 207). Apart from their alethic status, however, there are significant functional and formal similarities between Kant's *a priori* knowledge and Wittgenstein's logical and grammatical propositions.

30. How they constitute these possibilities is connected with Wittgenstein's account of objects which I discuss in section 3.1.1. There, I also discuss how Tractarian objects are connected with space which is based on a Neo-Kantian understanding of objects and manifolds (cf. section 2.2.1)

31. After all, he declares the propositions of logic "analytic" (TLP, 6.11), using Kantian terminology, but explicitly distinguishing logic from synthetic knowledge *a priori* which requires intuition.

32. By the same token, Kant declares Newtonian mechanics as necessarily true as it is the condition of possibility of natural science (in part III of the *Metaphysical Foundations of Natural Science,* Kant claims to "prove" Newton's laws).

2.2 The Formalisation of Geometry

2.2.1 Manifolds

In the middle of the 19th century, non-Euclidean geometries were developed in which at least one axiom (or postulate) turned out to be not necessarily true, namely the parallel postulate. What is important for the understanding of Wittgenstein's later employment of geometrical and spatial analogies is the increasing abstraction and formalisation of geometry, making it solely dependent on axioms, not on intuition. Since geometry was supposed to be the science of space, these developments naturally had serious ramifications in philosophy of space. But beyond that, it changed the status of *a priori* knowledge in general as Euclidean geometry had always been a model of absolute certainty and its method a prototype of scientific rigour and proof (cf. Reichenbach 1958, 8; Weyl 1919, 1). All this was threatened by the conceivability of various consistent geometries as developed by Bolyai, Lobachevsky and later geometers (cf. Bonola 1912).[33]

> Rather than being taken to be true of the world, pure geometry could now be taken to be a merely convenient tool that does not express any statement about the world. The demoting of geometry had a significant effect on what had been taken to be a priori knowledge, given that for Kant, space and time are given in intuition, forming the basis of geometry and arithmetic, respectively. (Stump 2015, 20)

Interestingly, these developments were embraced especially by empiricists such as Helmholtz and, later, by logical empiricists such as Schlick or Reichenbach. This can be explained by the following line of thought. Helmholtz argued against the Kantian view of the apriority of space because the developments in mathematics proved that Euclidean geometry cannot be derived from pure intuition: non-Euclidean geometries are conceivable which clearly contradicts the Kantian doctrine that the axioms of Euclidean space are apodictic and eternally true. Since we cannot know *a priori* which of the possible geometries is true, the nature of actual space has to be determined empirically. The distinction between pure geometry and applied geometry now becomes important. Pure geometry is a formal and mathematical science which allows for a number of possible axiomatic systems, none of which can be

33. The historical and critical study *Non-Euclidean Geometry* by Roberto Bonola summarises and comments on these developments and remained highly influential for decades (cf. Stump 2015, 33) which proves the relevance of the topic at the time. The concrete examples of consistent geometries without the parallel axiom are discussed in chapter 4 of that book. It was still a reference point in Wittgenstein's time: it underlies, for example, parts of Reichenbach's work *The Philosophy of Space and Time* (1928 [engl. 1958]), especially chapter 1. More recently Stump commented extensively on the development of Non-Euclidean geometry and the increasing formalisation of the discipline (2015, especially ch. 2).

said to be *a priori* true. Physical or applied geometry, by contrast, investigates the nature of actual, physical space and is, at least partly, *a posteriori*. Helmholtz made use of both insights: on the one hand he liberalised the notion of space in general and was thereby close to the kind of conventionalism that can be associated with pure geometry; on the other hand he suggested that physical space was to be constructed out of empirical, sensual data which he thought of as themselves organised in space-like structures which he called manifolds.

This idea marks an important point in the development of the spatial concepts with which we are concerned. The essential groundwork for this step was laid by Riemann who developed a theory of unspecified manifolds with *n* coordinates and with variable metrics of which three-dimensional Euclidean space was only a special case (cf. Riemann 2016). This very abstract notion of manifold lends itself easily to various applications:[34] Helmholtz drew on Riemann and developed an account of space-like manifolds to account for domains of sensory experience.

> Riemann calls a system of differences in which the individual element can be determined by n measurements, an n-fold manifold, or a manifold of n dimensions. Thus the space that we know and in which we live is a three-fold extended manifold, a plane a two-fold, and a line a one-fold one, as is indeed time. The system of colours also constitutes a three-fold manifold, in that each colour can be represented [...] as the mixture of three elementary colours, of each of which a definite quantum is to be chosen [...] We could just as well describe the domain of simple tones as a manifold of two dimensions, if we take them to be differentiated only by pitch and volume (Helmholtz 1903, 16–17, quoted in Hyder 2002, 26)

As Hyder observes, Helmholtz's theory is an important prerequisite for the *Tractatus* concept of logical space. Logic, he writes, "is, in a sense, a general theory of manifolds" (Hyder 2002, 24). Although it was primarily directed against Kant's doctrine of a unique and universal geometry, he also points to the Kantian elements of this "manifold theory of perception". Insofar as *a priori* forms of intuition constitute possible empirical intuition, Helmholtz's theory is "a modified form of Kantian epistemology of the sciences". Its basic feature is that

34. One of the thinkers who took up Riemann's ideas in a particularly prolific way was Albert Einstein who referred to Riemann's insight that a curved space might be boundless and yet finite (cf. Einstein 1920, §31). The theory of manifolds in which geometries with variable curvature can be determined by differential geometry, that is, by considering only parts of a geometry, not the whole, and establishing its metrics by means of geodetic lines (Bonola 1906, 137 ff.), is also the underlying structure of the Space-Time conception in general relativity whose curvature depends on the distribution of matter.

> all elements of human perception come structured in manifolds: colours, aural tones, sensations of hardness, warmth, etc. are all supposed to be organised in space-like manifolds. [...]
>
> It uses an expanded notion of *Anschauung* or intuition to characterise the field of "possible signs" that may make up our experience. To conceive of something is to conceive of all the possible representations that thing might bring about in us. (Hyder 2002, 19–20)

What is important here is the peculiar notion of *objects* that comes with this conception. While in traditional epistemology an object has spatial coordinates *and* properties, in the manifold theory all properties are represented by means of coordinates. A blue, heavy object at location [3,5,2] would have a more comprehensive "location" in a five-dimensional space with three spatial dimensions proper plus one dimension for colour and one for weight: [blue, heavy, 3,5,2] – blue and heavy could, of course, be quantified in a more exact manner, but here we are still talking about *perceptual* manifolds, not measuring manifolds; so this rough example should suffice to illustrate the idea. Anyway, the important point is that objects lose their "independence" in this view: they only exist through their relations to other points in the system to which they belong. Its assumed independent existence is replaced by the object's "address in the point-fabric" as Hyder puts it taking up a later remark from Wittgenstein's 1929 manuscripts (MS105, 70; Hyder 2002, 26). As there is no explicit reference to this theory in the *Tractatus*, Hyder presents his interpretation rather modestly as a background for the understanding of logical space; for a holistic understanding of Wittgenstein's space metaphors the similarity is in fact quite profound.

Due to their central role in the Tractarian system and the lack of concrete examples, the nature of "objects" is an important and controversial question in the interpretation of the *Tractatus*. Jaakko and Merrill Hintikka propose to understand them as phenomenological objects within a phenomenological language, which has been criticised by others as incompatible with the *Tractatus* and Wittgenstein's later reflections on it (cf. Kienzler 1997; Monk 2014). Hyder follows, again quite modestly, the phenomenological interpretation, but has better reasons to do so than the Hintikkas: he takes phenomenology in the sense of sensual perception that is structured in space-like manifolds. In this sense, it might be a good working model for the *Tractatus*, but this notion of phenomenological objects is very different from e.g. Russell's "objects of acquaintance" which the Hintikkas see as a precursor of Tractarian objects (Hintikka and Hintikka 1986, 51 ff.). In fact, the sense of objects as defined by coordinates in a multi-dimensional space is a very abstract conception that lends itself to many interpretations, phenomenological or physical, but there are strong indications that logical space is meant to be more general than those special spaces. I will later show how

the notion of logical space is related to the picture theory in which Wittgenstein distinguishes "spatial pictures" from "logical pictures". The former is obviously not used in the sense of logical space, but in a physical or geometrical sense which is less general than logical space:

> Each picture is also a logical one.
>
> (On the other hand, for example, not every picture is spatial) (TLP, 2.182).

Elsewhere, Wittgenstein uses manifolds that are space-like in the formal sense and very similar to Helmholtz's sensual manifolds: he speaks of colour-space, pitch-space and the like, but also of an infinite space which surrounds the spatial objects. Again, "spatial object" is used in a geometrical or physical,[35] not in a logical sense, and the "infinite space" surrounding it is a specific manifold which can be contrasted with other spaces:

> A spatial object must lie in infinite space. (A point in space is an argument place.)
>
> A speck in a visual field need not be red, but it must have a colour; it has, so to speak, a colour space round it. A tone must have *a* pitch, the object of the sense of touch *a* hardness, etc. (TLP, 2.0131)

So these specific spaces, it seems, must not be equated with logical space in which any kind of object can be represented. Wittgenstein considers manifold structures for different types of objects, physical and phenomenological, but he strives for an all-embracing structure, or rather: form, in which all possible configurations of any object can be represented. In the *Tractatus*, he is mainly concerned with the *logical* properties of what he calls "objects". In light of what has been said so far, we can conceive of these logical properties, in analogy to the sensual properties of objects in Helmholtz's theory, as locations in a general manifold that represents all possible relations, all possible states-of-affairs (*Sachverhalte*) in which an object can occur. The object is determined by its "logical coordinates" which are introduced as an analogy to physics in October 1914 and referred to repeatedly in the wartime manuscripts.

35. By referring to the *Prototractatus* (MS104, 2.0141), where Wittgenstein speaks indeed of "material points" in infinite space, Gerd Graßhoff argues, unlike Hintikka, for a physical nature of these objects in the sense of Hertz (1997, 116). There clearly is a similarity between Hertz's "position of a material point in infinite space" which is "conceivable" (*denkbar*; Hertz 1899, 48) and Wittgenstein's conceivable positions in logical space (see section 2.3 below). However, Hertz explicitly restricts his notion of conceivable to "geometrically conceivable" whereas Wittgenstein speaks about the wider, logical, sense of conceivable. "Infinite space" is just a three-dimensional sub-space of the multi-dimensional logical space So I think it is equally misled to conceive of *all* Tractarian objects as physical objects as it is to declare them all phenomenological; that a point is an "argument place" (PT 2.01411) highlights its dependency on a surrounding system or space, whose most general form is *logical*.

> We might conceive two co-ordinates a_P and b_P as a proposition stating that the material point P is to be found in the place (ab). For this statement to be possible the co-ordinates a and b must really determine a place. For a statement to be possible the logical co-ordinates must really determine a logical place! (NB, 29/10/1914)
>
> If a point in space does not exist, then its co-ordinates do not exist either, and if the co-ordinates exist then the point exists too. That is how it is in logic. (NB, 21/6/1915)

2.2.2 Geometrical and Epistemological Conventionalism

One could say that the development of various consistent geometries and the theory of manifolds forms the basis for conventionalism. This view has been associated with authors like Riemann, Helmholtz and Poincaré who had contributed to that development and argued for the equivalence of different axiomatic systems. However, there is a difference – of which these thinkers, but not all who were influenced by them, were aware – between conventionalism in pure geometry and in physical geometry. In the former, preference of any system can only be based on considerations of simplicity or convenience. Poincaré who recognised the equivalence of various kinds of geometries, expressed this view very clearly: that geometrical axioms can only be conventions and that empiricism has no role to play in geometry (cf. Poincaré 1913, 81 ff.). However, Reichenbach (like other logical empiricists, e.g. Schlick) misinterprets Poincaré by ascribing to him the same conventionalism when it comes to physical space (cf. Reichenbach 1958, 49). In fact, with regards to physics, Poincaré, just like Helmholtz, would regard empirical input as necessary. He would hold that particularly well-confirmed empirical regularities may be "elevated" to the status of conventions which then take up similar roles as axioms. They then constitute a space of possibilities while they themselves are normally not questioned when operating within this system and in this sense "necessary" (cf. Stump 2015, ch. 3: *Poincaré's Conventionalisms*). However, they are still conventions and not *a priori* truths in a Kantian sense, insofar as they are hypotheses which may change over time in light of contradicting evidence.

While Wittgenstein's later philosophy, especially his remarks on the foundations of mathematics, has often been considered in the context of conventionalism (e.g. Dummett 1978; C. Wright 1980; Shanker 1987), his early philosophy seems to be generally opposed to this doctrine. However, a certain proximity to conventionalist ideas can already be seen in the *Tractatus*. Stump thinks a lot about the apparent contradiction in Poincaré's view that principles are simultaneously "certainly true" and "neither true nor false" (Stump 2015, 45).

But this seeming contradiction is very close to Wittgenstein's view who argued, already in the *Tractatus*, that something which is certainly true, something which cannot possibly be false, cannot be called "true" in a strict sense. A logical truth "says nothing" and thereby is different from a normal, empirical truth which does tell us something about the world. This does not mean that Tractarian logic is a convention; after all it is supposed to be "the great mirror" of the world (TLP, 6.13). But the idea that something which constitutes the possibilities of empirical knowledge, which determines the candidates for truth, is not itself a kind of knowledge, but part of the symbolism, is a step away from Kantian necessary truth and towards a more conventionalist account as Wittgenstein's later philosophy might be labelled.[36]

I said earlier that Wittgenstein's view on science can be considered anti-Kantian insofar as it does not accept general laws such as Newtonian mechanics as *a priori* knowledge about the world. In fact, he is here, like Helmholtz, Hertz and Boltzmann, more Kantian than Kant, by holding that various scientific theories are *forms* of representation which do not describe the world in itself:

> the fact that it can be described by Newtonian mechanics asserts nothing about the world; but this asserts something, namely, that it can be described in that particular way in which as a matter of fact it is described. The fact, too, that it can be described more simply by one system of mechanics than by another says something about the world. (TLP, 6.342)

The Tractarian view on science is Kantian insofar as it acknowledges the boundedness of our faculty of knowledge and reason, the dependence of the world on the forms through which we approach them. It is anti-Kantian insofar as Wittgenstein would not accept those things for true which Kant held to be part of the *a priori* structure of the mind and thereby apodictic: Euclidean geometry and Newtonian mechanics. For this reason, it has been claimed that the Tractarian stance on science is "conventionalist in the vein of Hertz and Boltzmann" (Glock 1996, 343). I will investigate in the next section whether this claim can be upheld and how these physicists influenced Wittgenstein's concepts of picture and space.

36. Yemima Ben-Menahem explores Wittgenstein's ambivalent relationship with conventionalism. In the end, she argues against a foundationalist understanding of philosophy which holds conventions to justify our practice. However, she acknowledges conventionalist tendencies in Wittgenstein insofar as he does not see necessity as reflecting substantial truths, but as a matter of logic or grammar, that is of the forms of representation (cf. Ben-Menahem 2006, 256).

2.3 Space in Neo-Kantian Theoretical Physics

While scholars have for a long time sought to discover connections between Wittgenstein on the one hand and Boltzmann and especially Hertz on the other hand (Hacker 1986; Monk 1990; Graßhoff 1997), recent scholarship highlights the differences between them, focusing on Wittgenstein's originality and "creative appropriation" of their ideas. (Preston 2017, 121; cf. Aguilar 2015, 61–74; Pilch 2017, 21–23). I follow Hyder in that I do not claim any explicit and straightforward influence, but investigate a kinship between ways of thinking. After all, this is what even the greatest sceptics about Neo-Kantian influence admit: that Hertz and Boltzmann played a role in shaping the Wittgensteinian way of philosophising. We should also keep in mind Wittgenstein's list of influences in which the two physicists are mentioned first: "Thus Boltzmann, Hertz, Schopenhauer, Frege, Russell, Kraus, Loos, Weininger, Spengler, Sraffa have influenced me" (MS154, 15v).

The two key words that are associated with Hertz and Boltzmann in the context of the *Tractatus* are "picture" and "space". To be sure, neither of these ideas can be said to be a direct precursor of the Tractarian picture theory or logical space. The reason is simple: these terms were not elaborated to consistent theories by the physicists. Although they use the word "picture" in prominent places – in the quasi-Kantian way mentioned above – neither of them can be said to propose a picture theory.

Similarly Hertz's usage of space-like manifolds and models is restricted to the rather narrow context of his mechanics and cannot directly be applied to the logical space of the *Tractatus*. The so-called "phase space" that is often ascribed to Boltzmann – and whose foundations he has certainly laid – has striking similarities with Wittgenstein's logical space (cf. Janik and Toulmin 1973, 198; Glock 1996, 220), but cannot have influenced Wittgenstein directly as Boltzmann did not elaborate the idea which has only later been taken up by other physicists (cf. Pilch 2017, 21–22).

Given a) the mainly negative trend in the literature concerning the relation Wittgenstein-Hertz/Boltzmann, and b) my own focus on space, my approach is to trace the continuity from Kant via Helmholtz to these physicists to see what Wittgenstein might have found interesting in them. I will focus on Hertz since his influence is much more visible than Boltzmann's in Wittgenstein's writings; however, there are intersections in their respective views on picture and space which motivates putting them into one section. Both Hertz and Boltzmann used ideas by Kant and Helmholtz to provide representations of states of physical systems. Again the idea of spatial locations as *possibilities* is central. The application of these possibilities to physics can be seen as a result of the increasing abstraction of geometry in the 19th century: if "space" is no longer the concrete three-dimensional structure we live in, but an abstract

manifold of possibilities, then it can be used to represent other systems of possibilities as well.

Like Helmholtz, Hertz and Boltzmann are generally considered Neo-Kantians. What motivates this label is their acknowledgement of the Kantian insight that our knowledge is constrained by *a priori* structures which determine what can be considered possible within the framework; that we do not achieve any knowledge of "things in themselves", but only of things as they appear to us. In Kant, this comes with a claim for universality: Euclidean geometry is, as much as space is concerned, the only possible *a priori* form of intuition. Helmholtz, in light of the development of non-Euclidean geometries, denied this universality and extended the idea of *a priori* spaces to perceptual manifolds. Hertz and Boltzmann applied it to physical theories. In these theories, reality is not described directly and as such, but *represented* in what they called "images" (*Bilder*), scientific models that capture only aspects of the external world. In the preface to the *Principles of Mechanics*, Hertz introduces this terminology in a manner strikingly parallel to the introduction of Wittgenstein's picture theory (TLP, 2.1): "We form for ourselves images or symbols of external objects" (Hertz 1899, 1). This focus on representation, which is present in both Hertz and Boltzmann, is what is seen as the specific Kantian element in their thinking (cf. Janik and Toulmin 1973, 139 ff. Preston 2017, 112). To show what distinguishes this position from e.g. Mach's phenomenalism, Janik and Toulmin draw particularly on the difference between the German words *Darstellung* and *Vorstellung*, which were central terms in late 19$^{\text{th}}$ century Vienna. The latter has a connotation of passivity, of empiricist "impressions" and of private phenomenal knowledge, whereas the former is essentially public and highlights the active construction of a system by the scientist (cf. Janik and Toulmin 1973, 140). I will later argue that the difference between those words also entails a difference in terms of the modal status of what is represented (section 3.2.2); at this point Janik and Toulmin's distinction is sufficient to see the Kantian element in the Neo-Kantians' view on science. Their *Bilder* are not merely impressions as in empiricist idealism, but actively constructed models; *qua* models, however, they are also not straightforwardly realist, but only representations of aspects of reality. In his preface, Hertz goes on to formulate three criteria or requirements which these images should meet. First, a picture has to be *logically permissible*, that is, not contradicting the laws of our thought. Second, it should be *correct*, that is, be in agreement with our observations, be true. Third, it should be *appropriate* which is specified as satisfying the values of distinctness and simplicity (Hertz 1899, 2). The particular image Hertz is going to suggest is meant to be more appropriate than other permissible and correct systems of mechanics, for example the Newtonian one.

With these considerations in mind, we can go back to the question of conventionalism. Preston argues that there might be some motivation to ascribe conventionalism to both Hertz and Boltzmann. Hertz's claims, for example, are restricted to only one aspect of reality: the form of an image "is such that the necessary consequents of the image in thought are always images of the necessary consequents in nature of the things pictured" (Hertz 1899, 1). The second book of the *Principles of Mechanics*, however, centres on the "fundamental law" which is supposed "to embody the entire empirical content of his system and thus", according to Preston, "is a poor candidate for being a convention". And Boltzmann, he continues, can even less be called conventionalist "since the images he has in mind, although there are some arbitrary aspects, are genuine hypotheses" (Preston 2017, 115). Without engaging too much in a terminological debate, we can simply state that this notion of conventionalism is very different from both variants we encountered in the last section, and presumably also from the way Glock had intended his statement. Even if Hertz considered his image to be, in a sense, complete and to account for the totality of empirical content, it is nevertheless and very explicitly considered *one* possible image among others. It might be more "appropriate" than Newtonian mechanics, to which it is compared in the preface and which it is supposed to simplify by eliminating the concept of force; but preference in virtue of simplicity is precisely what one can call a conventional choice. Given Hertz's three requirements for images, one could regard the first criterion as compatible with Poincaré's geometrical conventionalism: here indeed all systems that are free from contradictions are equivalent. The second criterion, the correctness of the image, can be compared to Poincaré's view on scientific principles in which empirical hypotheses, if they prove to be sufficiently "correct", may be elevated to the status of principles which then constitute a scientific image or theory.[37] If conventionalism is understood in this way, Preston's argument against ascribing conventionalism to Boltzmann loses its plausibility, too: the images being hypotheses does not exclude them being conventions. Finally, Hertz's third criterion, appropriateness, may indeed be considered a guide in choosing one convention among those that meet the first two criteria.

Why this discussion about pictures and conventions? Because pictures are often considered an important connection between Hertz and Wittgenstein. In contrast, I think their respective adoption – and rejection – of elements of Kantian philosophy is more important and this is related to their respective treatment of space as *a priori* form of empirical knowledge. Kant's position towards the status of our knowledge is somewhat ambivalent. On the

37. Sometimes Hertz distinguishes images from "scientific representations of images" (Hertz 1899, 2), but mostly he uses it quasi synonymously to "system of principles" or scientific theory, especially when he compares his image to Newtonian mechanics and early electrodynamics (e.g. 40). It is probably fair to say that the term is not sufficiently well-defined in the preface to ascribe a picture-*theory* to Hertz (cf. Aguilar 2015, 57–58).

one hand he distinguishes between thing in itself and thing as appearance, the former being inaccessible to human knowledge, the latter being constituted by our *a priori* schemata. It shares with conventionalism the idea that reality in itself cannot be known. On the other hand, he was quite anti-conventionalist in his own theory of space and science since he only allowed for *one* spatial structure and assumed necessary laws of mechanics, namely Euclidean space and Newtonian mechanics, which are both *a priori* certain and not conventional.[38] The liberalised notion of *space* after the development of non-Euclidean geometry and the abstract theory of manifolds enabled the Neo-Kantians to be both Kantian and conventionalist where Kant himself was universalist; they accepted *a priori* structures in a weak sense of *a priori* which determine what kind of knowledge the system can provide, but also the plurality of such systems. The choice of one system of images over another may then have empirical reasons – Helmholtz, Hertz, and Boltzmann were empiricists – but never to the extent that one system could count as the only accurate description of reality in a metaphysically realist sense. The emphasis on simplicity and clarity in Hertz's third requirement, in fact demonstrates the proximity to Poincaré's conventionalism in which simplicity is the decisive factor for his preference of Euclidean geometry.

The relation of Hertz's images to Wittgenstein's pictures is rather loose in any case.[39] Remark 2.1 of the *Tractatus* might be intended as a conscious echo of Hertz's preface, but the difference is no less obvious than the similarity: for Hertz, "we make to ourselves pictures of objects" while for Wittgenstein, "we make to ourselves pictures of facts" – and Wittgenstein highlights the difference between objects and facts repeatedly in the *Tractatus*, e.g. in the second sentence of his book (TLP, 1.1). But even granted that the use of "objects" in Hertz is to be understood as a sort of *pars pro toto* for the external world – after all, he was less concerned with logic than with physics where it is kind of natural to start with "objects" – even then the further use of *Bild* is rather different from the representation of facts by means of propositions as in Wittgenstein's picture theory. Hertz' images are themselves something like theories and certainly not concerned with propositions as pictures of facts.

The reason for comparing theories to spaces and these to the various connected concepts in Wittgenstein is the constitutive function which these frameworks have over its content, that is, not the empirical content, but what can be said *a priori* about, in this case, mechanics. It is also what Hertz stresses when he explains how he will understand the term "principles" in his book:

38. Poincaré introduced his notion of convention precisely to distinguish this specific type of non-necessary constitutive principle from Kantian *a priori* knowledge (cf. Ivanova 2015, 118).

39. Both use the German *Bild*, but it is perhaps no accident that in the translations different words are used.

> by this will be meant any selection from among such and similar propositions, which satisfies the requirement that the whole of mechanics can be developed from it by purely deductive reasoning without any further appeal to experience. In this sense the fundamental ideas of mechanics, together with the principles connecting them, represent the simplest image physics can produce of things in the sensible world and the processes which occur in it. By varying the choice of the propositions which we take as fundamental, we can give various representations of the principles of mechanics. Hence we can thus obtain various images of things; and these images we can test and compare with each other in respect of permissibility, correctness, and appropriateness. (Hertz 1899, 4)

For this is what interests me in the background of all these "Kantian" notions of space. Principles, geometrical, theoretical, grammatical, are meant to form a framework from which corollaries can be deduced and which constitute possibilities *within* the framework; hence something like *a priori* structures, spatial structures.[40] Hertz recognised that these principles keep their constitutive function, even if it should turn out that they must be modified in light of contradicting experience. In fact, he deliberately designed his "image" to satisfy this requirement:[41]

> The correctness of the image in all cases was carefully provided for by making the reservation that, if need be, facts derived from experience should determine definitions or *vice versa*. (9)

Even though this is certainly not true in the case of Wittgenstein's logical space (it is to an extent in his later work), I see formal similarities in Hertz's "image" and Wittgenstein's logical space, more than between their respective "picture theories". Let me elaborate this.

The full title of Hertz's book is *The Principles of Mechanics Presented in a New Form*. This new form, the form that distinguishes his "image" from the others considered in the preface, is a mathematical form that is characterised above all by its *systematic* approach to mechanics: his starting point are not points, but "systems of points" (29). Using the term manifold in the sense of his teacher Helmholtz, he explains the relation of this method to geometry.

40. An idea that comes up in Schopenhauer and in Wittgenstein is that there is, in fact, no difference between axioms and corollary: both are equally well – or equally badly – founded. If a statement follows necessarily from one or more axioms, then its necessity is not smaller than that of the axiom itself (cf. Schopenhauer 2008, 109). And Wittgenstein in the *Tractatus*: "All propositions of logic are of equal rank; there are not some which are essentially primitive and others deduced from there. Every tautology itself shows that it is a tautology" (TLP, 6.127).

41. This is similar to Reichenbach's later suggestion to keep the constitutive aspect of Kant's *a priori*, but to dismiss its universality (cf. Reichenbach 1965, 48).

> A system of n points presents a $3n$-manifold of motion – although this may be reduced to any arbitrary number by the connections of the system. Hence there arise many analogies with the geometry of space of many dimensions; and these in part extend so far that the same propositions and notations can apply to both. (30)

These are formal analogies, writes Hertz, emphasising the differences between pure geometry and physics. It is indeed important to distinguish two notions of geometry in his book: the physical space he deals with is Euclidean as he makes clear in the very first explanations of space, time and mass (45); in this regard Hertz is a strict Kantian. However, with regards to his method, he makes use of the abstract account of geometry as delineated in this chapter. Why does he use the method of formal geometry which he regards as completely detached from actual physical space? Because "it enables us to render the most general and comprehensive statements with great simplicity and brevity" (31). Jesper Lützen who argues that the "geometrization of mechanics" is the main innovation of the *Principles of Mechanics* (2005, 159) provides an explanation why Hertz insists on the Euclidean nature of physical space and hesitates to admit the geometrical background although his own theory of systems of points is very close to Riemannian formal geometry:[42]

> In modern textbooks a Riemannian manifold is presented as a formal analytical system to which are attached geometric names such as point, distance, map, atlas, curvature, etc. Therefore we have no problem in identifying Hertz's geometry of systems of points with a Riemannian manifold. However, one has to keep in mind that in Hertz's time the theory of Riemann's geometry was totally entangled with a discussion of physical space.
>
> [...] It is symptomatic that although Hertz introduced many other geometrical terms in his geometry of systems of points, he never used the term *point* for his systems of points nor the term *space* for what we today call configuration space. Thus Hertz could benefit from the formalism of high-dimensional Riemannian manifolds, without using the philosophical [*sic!*] problematic concept of a high-dimensional non-Euclidean *space*. To Hertz, the relation between his geometry of systems of points and the Riemannian geometries introduced by the mathematicians was not one of equality or inclusion but one of analogy. (Lützen 2005, 155)

42. "He [Hertz] clearly accepted high-dimensional geometries with variable curvature. His own theory of systems of points is precisely that." (Lützen 2005, 130).

By "analogy" he means the formal analogies mentioned above which I take to be also the main point of comparison to Wittgenstein's *Tractatus*. As Lützen points out, Riemann and Hertz's systems "have entirely different objects, but shares [*sic!*] the analytic formalism" (Lützen 2005, 156). This is also true of Wittgenstein's system: its objects are logical objects of an entirely different nature, but the formalism of logical space indeed resembles the highly abstract, multi-dimensional systems of Riemann and Hertz. One can even apply Hertz's words to Wittgenstein that these analogies "in part extend so far that the same propositions and notations can apply to both". Indeed, the most obvious influence of the *Principles of Mechanics* on the *Tractatus* is on the level of terminology. As already said, the use of *Bild* is quite different from Hertz, but there are more similarities. There seems to be a connection between Hertz's "Lage" (position) and Wittgenstein's "Sachlage" which both refer either to a location or to the state of an entire system (cf. Hyder 2002, 130); moreover, that these possible positions in infinite space are "geometrically conceivable" (Hertz 1899, 48) echoes in Wittgenstein's conception of logically conceivable positions in logical space; this shall be discussed in section 3.1.2 in the next chapter. The term "coordinates" is used in both works as coordinates in an abstract multi-dimensional manifold (49) and in Wittgenstein's *Notebooks* they mark the beginning of all space analogies (cf. Pilch 2017, 17).

> The internal relation between the proposition and its reference, the method of symbolizing is the system of co-ordinates which projects the situation into the proposition. The proposition corresponds to the fundamental co-ordinates. (NB, 29/10/1914)

Furthermore, Wittgenstein adapts Hertz's term "coordinates of configuration" in the *Tractatus*, but for considerably different purposes. In TLP 2.0231 he uses the term "configuration" in "configuration of objects", establishing the important relation between simple objects and Sachverhalte: "The configuration of objects produces states-of-affairs [*Sachverhalte*]" (TLP, 2.0272). And this configuration as a changeable matter is contrasted with the stable persistence of objects which are "indestructible" for Hertz and Wittgenstein alike (Hertz 1899, 46; TLP, 2.0271; TS213, 98r).

However, there are also considerable differences: while the respective notions of "coordinates" have in common that they specify locations in a geometrical structure, Wittgenstein's "logical coordinates" do not concern (logical) positions of simple objects, but rather the position of an entire *Sachverhalt*. (This object-fact distinction corresponds to the differing concepts of pictures). Another difference lies in the fact that Hertz's whole enterprise is a theory of dynamics which shall provide a framework for describing the movement of particles; in Wittgenstein's space nothing moves and it is a remarkably timeless structure that is explained here (cf. Pilch 2017, 23). More shall be said on this in section 6.1.

The aim of this section was to highlight the role of the formalisation of geometry in the works of those physicists Wittgenstein is known to have absorbed. While the focus has been on Hertz's geometrical method, something similar can be said of Boltzmann's whose influential ideas in statistical physics are similarly based on the notion of multi-dimensional abstract manifolds in which possible states of systems can be modelled. While the relation between this "phase space" and Tractarian logical space may be "no more than a loose analogy" (Preston 2017, 118), the formal instruments are certainly shared. With regards to Hertz, it might well be that his focus on a perspicuous presentation by means of geometrical methods while leaving the facts untouched has influenced Wittgenstein on a more profound level than generally assumed. While I am sceptical of comparisons with regards to pictures or objects, I think with regards to the methodological usage of space, there is an intriguing similarity between these thinkers.

2.4 Summary

To sum up, what we should bear in mind when discussing logical space in the *Tractatus* is the increasing abstraction and formalisation of space. For Kant, space was a pure form of intuition and thereby a framework of possibilities for empirical intuition. With the development of non-Euclidean geometries his claim for necessity and universality was no longer tenable; Riemann and Helmholtz abstracted and extended the notion of geometry to unspecified, mathematical manifolds of variable curvature and dimensionality. This enabled something like geometrical conventionalism whose most important proponent is Poincaré who sees the various geometries that are possible in such a manifold as on a par: each is *a priori*, but precisely for this reason it is no longer necessary. However, when considering physical space, the choice of geometry is constrained by empirical input. Necessity in this sense amounts to something like probability, particularly well-confirmed empirical regularities may be elevated to the status of laws. There is still an element of convention here, but it is less free than geometrical conventionalism.

As to Hertz and Boltzmann, I agree with the current trend in the literature that a strong influence in terms of concrete philosophical theses cannot be found; rather, they contributed to Wittgenstein's particular style of philosophising. However, some aspects of Hertz' thinking do have a strong echo in the *Tractatus* and beyond. His division into thinkable (conceivable) and unthinkable shall guide Wittgenstein's space analogies from "logical space" to "spatial grammar" and "language-game". More specifically, that states-of-affairs can be described as configurations of objects resembles strongly the configuration space in Hertz, although the particular use in the *Tractatus* is certainly a creative appropriation of the concept. And above

all, the use of geometric analogies to clarify systems that need not necessarily be described with these geometric methods is a typical characteristic of Wittgensteinian thinking as we will see.

It shall not be missed out that two rather important influences on the *Tractatus* have only been mentioned briefly so far: Frege and Russell. They are not only mentioned in Wittgenstein's retrospective list of influences (whose order is arguably important: Boltzmann, Hertz and Schopenhauer come *before* Frege and Russell), but also in the preface of the *Tractatus*. Although they commented on geometry as well, they do not take that much space in my investigation because they both seemed to have a rather conservative view in these matters. Stump mentions their respective arguments with Hilbert and Poincaré (cf. Stump 2015, 49). Of course, Russell was aware of the potential conventionalism of geometries – it was the topic of his fellowship dissertation – but he apparently was still very realist about physical space and could not anticipate the developments of general relativity. Although Wittgenstein was closer to Frege and Russell when writing the *Tractatus*, his later conceptions of space, parallel to his gradual alienation from Russell, seem to be closer to their opponents.

Chapter 3

Logical Space in the *Tractatus*

Stell dir Ordnung vor [. . .] aber jetzt stell dir bloß eine ganze universale, eine Menschheitsordnung, mit einem Wort eine vollkommene zivilistische Ordnung vor: so behaupte ich, das ist der Kältetod, die Leichenstarre, eine Mondlandschaft, eine geometrische Epidemie!

Robert Musil
Der Mann ohne Eigenschaften

In the last chapter, the Kantian notion of space as an *a priori* form of intuition has been spelled out and special emphasis has been put on this form's constitutive function: it determines the possibilities of empirical knowledge, but, by being a *pure* form, does not say anything about actual empirical facts in the world. In light of mathematical developments in the 19th century the Kantian view on geometry was modified by thinkers such as Riemann, Helmholtz and Poincaré who turned pure geometry into an abstract manifold whose dimensions and axioms can be defined by convention; physical space, however, was conceived an empirical question. Drawing on these developments, Neo-Kantians such as Hertz and Boltzmann utilised space-like structures to model the possible distributions of particles in physical systems.

These developments form an important basis for the understanding of Wittgenstein's *logical space* in the *Tractatus* as has already been indicated in the respective sections in the last chapter; an excellent account of these historical connections can be found in David Hyder's study of physical and mathematical space-concepts which can serve as a background

for my discussion of logical space (cf. Hyder 2002). I emphasise those aspects that are important for my overall claim: that the concepts of space and geometry play a profound role in Wittgenstein's work as a whole and can thereby provide a better understanding of continuity and discontinuity in his philosophy. This has been the topic of endless debates, mostly centring around the question how many "Wittgensteins" there are and if and how the later philosophy is superior to the *Tractatus*.[43]

In order to assess continuity or discontinuity with regards to space and geometry, it is necessary to carve out the features and functions of logical space in the *Tractatus* which can then be compared to occurrences of space in the middle period. In light of the similarities that will thus become evident, it will then be possible to see which elements remain vital in Wittgenstein's philosophy when the notion of space stops being as central as it is in the *Tractatus* and in the period 1929–32. While the connection between logical space and grammatical space in the middle period has been recognised by a number of scholars,[44] their similarity has not been spelled out and it has not been made clear what motivates the usage of the common term "space" – which is itself a highly controversial term in philosophy. In Wittgenstein, it seems to be used either metaphorically or in a very specific technical sense. Without specifying the continuity from logical to grammatical space, it is difficult to see the further continuity from grammatical space to language-game in the later philosophy which I shall argue for.

An account of the historical development of logical space in the war-time manuscripts has been given by Martin Pilch. He shows how Wittgenstein's idea evolves in a period of intense work in the first months of the war: the notion of "logical place" goes back to an early intuition in October 1914 about the analogy between a geometrical place and a proposition which both refer to a possibility; shortly after this, Wittgenstein was concerned with the relation between a proposition and its negation which turned out to be representable in a spatial structure, too; the completeness of logical space was a requirement that had to be met in order to make this account of negation work and to ensure that the spatial understanding of possibility is exhaustive (Pilch 2017, 17–20).[45] While this reconstruction is based on the traces of the thinking process in the wartime manuscripts, in the final version of the *Tractatus*, logical space is given important positions within the system of decimal numbers.[46] It is

43. See Kienzler 2017 for a summary of various accounts that have been proposed.

44. E.g. Glock 1996, 223; Hyder 2002, ch. 7; Ometiţă 2018, 150–151; Schulte 2017, 319.

45. Pilch also provides an instructive interpretation of the various functions of logical space by means of three representation spaces which correspond to Wittgenstein's usage of the terms "truth-possibilities", "truth-conditions" and "truth-arguments". I will draw on these illustrations in the respective part of my own account of logical space (see section 3.2).

46. I agree with Luciano Bazzocchi's view on the numbering system (cf. Bazzocchi 2010): remarks with fewer decimal numbers have a higher logical weight and a series of remarks with the same number of decimals

mentioned in a remark on the very first proposition: "The facts in logical space are the world" (TLP, 1.13). It occurs in the first part of the picture theory (TLP, 2.11; 2.202), and in the treatment of tautologies and contradictions (TLP, 4.463). In between, a whole one-decimal section is dedicated to logical space, namely TLP 3.4 which is composed of lower numbered remarks from the *Prototractatus* (MS104, 3.2101-3.2104).

Of course not only Hyder and Pilch, but numerous other scholars have recognised the significance of logical space within the system of the *Tractatus* (e.g. Anscombe 1959; Black 1964; Glock 1996; Metschl 2001; Sullivan 2001). What distinguishes my approach is its "holism" with regards to Wittgenstein's philosophy. My motivation is not only a certain curiosity about this strange concept and its role in the *Tractatus*. Rather, I see those features that logical space is supposed to illustrate as central characteristics of Wittgenstein's thinking as a whole, at all stages; therefore I emphasis those parts of the early ideas that shall be relevant in the modifications after 1929. Since my reading is largely compatible with the established accounts, I hope it will not be considered Whiggish, but recognised as an attempt to tackle the topics from a fresh angle. It is not only a particular angle to logical space, but to the *Tractatus* as a whole, so that my approach is doubly holistic in seeing the *Tractatus* as part of the whole oeuvre and seeing logical space as part of the whole *Tractatus*. In both respects, there is a more than local significance to the usage of space. While the significance of the space metaphor for Wittgenstein's whole philosophy, can naturally only be assessed over the course of my overall argument, this chapter aims to clarify the eminent role of logical space within the *Tractatus*. I do this by showing its relevance for three central themes of the book: possibility, "logical constants", especially negation, and the claim for universality.

The focus on these three themes under the aspect of space leads to contributions to some controversial issues. The question about the relation between *Tatsache*, *Sachverhalt*, and *Sachlage* can be addressed and illuminated, albeit not finally settled, within this framework: taking the risk of an awkward phrasing, I propose to indeed understand all three terms in the sense of possibilities while keeping their generally "factive" nature. Another question concerns the status of objects which, unlike the three factive structures just mentioned, are fixed and necessary, not contingent and changing. By distinguishing different readings of "possibility", I aim to elucidate the apparent paradox that objects are necessary by being possible as opposed to actual, while facts by being actual are contingent, that is, possible as opposed to necessary. Moreover, the question whether objects have to be understood phenomenologically or physically can be answered by pointing out the abstractness of logical space, which does not prefer any particular interpretation.

(e.g. 2.11, 2.12, 2.13 . . . 2.19) can be read as a continuous argument which as a whole is a comment on the respective "higher" remark (in my example: 2.1).

Wittgenstein's views on negation and other logical operations are intimately connected to the differentiated notion of possibility which I elaborate in the first section. A certain kind of possibility, namely the logical *form* of a picture, which corresponds to the possibility of a certain atomic fact, cannot itself be said by means of the picture, but only shown. Based on the spatial perspective, I advance an interpretation of Wittgenstein's "fundamental thought": logical space complements the idea of logical pictures as the basis of representation; "logical constants do not represent" because they have no "location" in logical space, but consist in transitions from certain areas in this space to other areas.[47] That such a transition is possible shows itself in the "internal relations" of the respective propositions which are expressed in the structure of the space: for example, a proposition and its negation cover complementary regions in logical space and a proposition following from another proposition falls completely within the region of the latter.

This approach also allows an interpretation of "nonsense" in the *Tractatus* that does not ascribe any substantial quality to it, but still allows it to be instructive insofar as nonsense can illuminate the logical space without being itself expressible *within* the space.[48]

The third section will investigate the *Tractatus*' tendency to absolute and general claims about propositions, logical space and the world. Some aspects of this view remain intact during the revisions of the middle period, especially the completeness of logical space which will still be important in Wittgenstein's later grammatical spaces and language-games. The universality and uniformity of logical space, however, seems to be an unfounded, or at least confused, postulation as Wittgenstein will recognise later on.

3.1 Logical Space: A Space of Possibilities

Potentiality dominates the terminology of the *Tractatus* – and with it, to use a problematic word, its ontology: what and how *is* a possibility? – to such an extent that a form of "modal realism" has been ascribed to it (recently by Hacker 2017, 214). Coined by later philosophers, notably by David Lewis, this term can be misleading, but the motivation to use it is understandable. I suggest distinguishing three senses of possibility to clarify how the term and related concepts are to be understood, especially with respect to its relation to logical space. My distinction follows a method that Wittgenstein repeatedly used himself in

47. The "one logical constant" which Wittgenstein admits (TLP, 5.47), the general form of the proposition, is itself intimately connected with the structure of logical space as I will argue in sections 3.2 and 3.3.

48. This has some similarities with Oskari Kuusela's interpretation of the *Tractatus*; he approaches it under the perspective of notation (cf. Kuusela 2019a). How space and notation are similar shall be shown in the course of this thesis, especially in section 5.2.2.

his discussions of various spaces in his later work, namely to ask: what is the opposite of the concept you use?[49]

1. Possibility as opposed to actuality ("merely possible")
2. Possibility as opposed to necessity ("contingent")
3. Possibility as opposed to impossibility ("thinkable")

The three senses do not necessarily exclude one another. Rather, they provide connected criteria to talk about facts, thoughts or pictures, not unlike Hertz's criteria for his scientific pictures which I mentioned in the last chapter: first a picture has to be "logically permissible", i.e. possible in the sense of (3), then it can be "correct", i.e. actual as opposed to "merely possible" in the sense of (1); Hertz's third criterion "appropriateness" becomes more important in Wittgenstein's later philosophy. For the current purpose, the three-fold distinction will suffice to illuminate a couple of issues. I start with an investigation of Wittgenstein's ontology where the relation of (1) and (2) is important; the more we get towards the *representation* of the world in language, the focus shifts towards (2) and (3).

3.1.1 Facts and Objects

Since the completion of the *Tractatus*, readers have wondered what the difference between *Tatsachen* and *Sachverhalte* might be.[50] Even the two philosophers whose work Wittgenstein called an important stimulation for his book, and to whom he sent a copy of the manuscript shortly after completion, struggled to make sense of this distinction: "Was der Fall ist, die Tatsache, ist das Bestehen von Sachverhalten" – "What is the case, the fact, is the existence of atomic facts" (TLP, 2).

> I take it that every fact is the existence of a *Sachverhalt* in such a way that a different fact is the existence of a different *Sachverhalt*. Now could we not cancel the words "the existence" and say: "Every fact is a *Sachverhalt*, every

49. At all stages of his philosophy the possibility of an opposite is a requirement for the meaning of a concept which is constituted by its *place* within a system – one place as opposed to others. See also section 3.2.

50. The translation of "Sachverhalt" is controversial and part of the problem: David Pears and Brian McGuinness tried to distinguish it from "fact" by translating it as "state of affairs" instead of "atomic fact" which was used in the first translation by Ramsey and Ogden. Since Wittgenstein commented on many other issues and thereby influenced the translation, I take it that Ogden's edition comes close to what Wittgenstein had intended. Therefore, I will mostly quote from this translation, but I will take into account the modifications by Pears and McGuinness which often seem to make more sense in English (cf. TLP 1961). One should also note, that Ogden's edition is bi-lingual and that therefore the translation could always be compared with the original which is what I will do especially with regards to controversial terms such as *Sachverhalt* and *Sachlage*.

> different fact is a different *Sachverhalt*"? Could we perhaps also say: "Every *Sachverhalt* is the existence of a fact?" You see: I get caught up in doubts over what you want to say [...] Are there also *Sachverhalte* that do not exist? Is any combination of objects a *Sachverhalt*? (Frege to Wittgenstein 28/6/1919)
>
> What is the difference between Tatsache and Sachverhalt? (Russell to Wittgenstein, 13/08/1919)[51]

Wittgenstein's reply to Russell reduces the difference to a matter of correspondence to elementary propositions and concatenated propositions respectively:

> Sachverhalt is, what corresponds to an Elementarsatz if it is true. Tatsache is what corresponds to the logical product of elementary props when this product is true. (Wittgenstein to Russell, 19/08/1919)

Wittgenstein highlights the simplicity of *Sachverhalt* as opposed to facts which is the obtaining of (one or more) *Sachverhalte* ("what corresponds to the logical product of elementary props"). The question of potentiality is not addressed, although it seems to be suggested by the choice of words in the book. If a *Sachverhalt* only corresponds to an elementary proposition "if it is true", what corresponds to an elementary proposition if it is not true? Why is it always emphasised that the truth of an elementary proposition and indeed the existence of a *Sachverhalt* does not matter, but only their possibility? Why is a *Tatsache* "the existence" (*das Bestehen*) of *Sachverhalte*? Is a *Sachverhalt* only a *complex*, a possible assembly of objects, while a *Tatsache* is the fact *that* things are arranged in a specific way? For Erik Stenius, a *Sachverhalt* is "something that could *possibly* be the case, a Tatsache something that is *really* the case" (Stenius 1960, 31); and Max Black considers a number of arguments for what he calls the "P-theory", that is, the assumption that *Sachverhalte* are to be understood as possibilities as opposed to facts which are always actual. More recently, Pilch argued for a distinction along these lines in his study of logical space:

> The difference is that while Sachlagen correspond to a possibility, Tatsachen point to reality. Tatsachen are defined as the "obtaining" (or "existence") of Sachverhalte (TLP 2). Logical space as presented at the beginning of the Tractatus is a totality of Sachlagen, the obtaining or not obtaining of all the Sachverhalte ("states of affairs") – some of these situations obtain (or exist) in reality and their obtaining is then called a "fact". (Pilch 2017, 24)

51. All letters are quoted from the *Innsbrucker Elektronische Ausgabe* (2004). The German originals I translated myself.

This is also what motivated Pears and McGuinness to translate *Sachverhalt* as "state of affairs" instead of "atomic fact"[52] which seems to imply that *Sachverhalte* are always actual which would render expressions like "möglicher Sachverhalt" very awkward: what is a possible or even a non-existent fact?[53] This latter view, that *Sachverhalte* are to be understood as facts, *atomic* facts, is what Black calls the "F-theory" which he holds to be preferable in the end (Black 1964, 38–45). One of his arguments is that "möglicher Sachverhalt" would be pleonastic if it was conceived as a possibility by definition. – So it seems that the idea of possible *Sachverhalte* stands in need of clarification either way.

Biographical evidence supports the F-theory: there is Wittgenstein's approval for Ramsey's translation,[54] there is his above quoted letter to Russell which seems likewise to suggest a difference of *simplicity*, not of modality, and which Russell used in his introduction (TLP, xi). A note by Ramsey, which refers to his discussions with Wittgenstein in 1923, points to the same direction and confirms that his translation of *Sachverhalt* as "atomic fact" was deliberate and probably used in agreement with Wittgenstein himself:

3 words for fact		atom prop (= W's elem prop)
Atom Fact	Sachverhalt	p
Fact	Tatsache	$p \cdot q$
State of Affairs	Sachlage	$p \vee q$

(Ramsey 2019, 002-29-01, 5)

This seems to suggest that all three terms are indeed factive, but distinguished by their being atomic, concatenated or connected by other operators.[55] How could Wittgenstein have failed to convince Russell and Ramsey that *Sachverhalte* are facts *in potentia* if this was indeed his intention? And how could he have agreed to the publication of an edition which did not acknowledge this difference? On the other hand, McGuinness's intuition that there is an element of potentiality in the notion of *Sachverhalt* is obviously justified, too, since they can "exist or not exist".

By distinguishing between possibility as opposed to actuality and possibility as opposed to necessity, this tension can be overcome. In line with ordinary usage, we can say: *Sachverhalt*, *Tatsache* and *Sachlage* are all "factive", they exist if and only if they exist. Possibility comes

52. Cf. McGuinness 2017, 86.

53. However, Wittgenstein does mention "negative facts" as the non-existence of *Sachverhalte* (TLP, 2.06). And in the wartime manuscripts he struggles with precisely this point: "It is the *dualism*, positive and negative facts, which gives me no peace. For such a dualism can't exist. But how to get away with it?" (NB, 25/11/1914)

54. The absence of correction can be counted as approval as Wittgenstein did correct many other things in the translation and gave extensive advice to the editor (cf. Wittgenstein to Ogden 23/4/1922).

55. This three-fold distinction agrees, by the way, with Pilch's three representation spaces of logical space: his parameter space accounts for *Sachverhalte*, state space for *Tatsachen* and propositional space for *Sachlagen*. I think, he is not committed to the "P-theory". See illustrations of these representation spaces in 3.2.

in not by their being not actual ("A logical entity cannot be merely possible", TLP 2.0121), but by their being contingent. Each fact, even if it exists, could just as well not exist; it is contingent, that is, not necessary. And this possibility is what Wittgenstein's investigation of logic and language is about. Logic is, in a way, not concerned with facts – this is what concerns the natural sciences – but with the possibilities of facts: "all possibilities are its facts" (TLP, 2.0121). Language has the capacity to present a fact even if it does not exist, hence it does not refer to its existence, not to the fact itself, but to its "existence or non-existence": the possibility that things are arranged in such a way as they are actually, *in fact*, arranged in the fact.

This brings in the notion of things. Initially, objects are surprisingly stripped of their traditional role as building blocks of the universe: "The world is the totality of facts, not of things" (TLP, 1.1). However, they are soon introduced as the constituent parts of atomic facts[56] (TLP, 2) followed by a number of comments highlighting their modal status with regards to atomic facts. While facts are contingent, objects are necessary. Facts can obtain or not obtain, while objects already contain all possibilities of being part of a fact. Which facts obtain is contingent, is, in Tractarian terminology, "external" to the object; but it is an "internal" property of an object to be capable of being part of these facts. Therefore, objects are not *actual*, as facts are, but they are *necessary* precisely by comprising all possibilities of actualities. The cast of a die can result in six different "facts" each of which is contingent, but that there are these six possibilities is a necessary property, an internal property of the object dye.[57] This transition from possibility to necessity, from *can* to *must*, is particularly obvious in remarks 2.012 and 2.0121.

> In logic nothing is accidental: if a thing *can* occur in an atomic fact the possibility of that atomic fact must already be prejudged in the thing. (TLP, 2.012)
>
> It would, so to speak, appear as an accident,[58] when to a thing that could exist alone on its own account, subsequently a state of affairs could be made to fit.
>
> If things can occur in atomic facts, this possibility must already lie in them.
>
> (A logical entity cannot be merely possible. Logic treats of every possibility, and all possibilities are its facts.) (TLP, 2.0121a—c)

56. Speaking of part and whole is, of course, problematic since atomic facts are atomic: they are a *combination* of objects (TLP, 2.01) which are of a different ontological category. I hope to make clear in this section that "existence" of an object is completely different from "existence" of an atomic fact, for which Wittgenstein prefers to use the word "Bestehen" (obtaining), and both from a concrete statement such as "Socrates existed".

57. Sluga, in a talk in Norwich, made an interesting etymological argument by highlighting that both the German word "der Fall sein" and the English "to be the case" go back to the falling of dice as an epitome of contingency. The expression in TLP 1 thereby entails both facticity and contingency.

58. The words "zufällig" and "Zufall" take up the etymological implications of "der Fall".

Wittgenstein makes it clear that logic is concerned only with necessities, and it gets this necessity by regarding *all* possibilities of contingent facts. That also the "world" is to be understood in this sense has already been expressed implicitly in the strange addenda to proposition 1.1: that *all* facts form the world (TLP, 1.11), that this determines also what is not the case (TLP, 1.12) and that these are the facts in logical space (TLP, 1.13).

I deliberately quoted only half of proposition 2.0121 because the second half elucidates the point from a different angle:

> Just as we cannot think of spatial objects at all apart from space, or temporal objects apart from time, so we cannot think of *any* object apart from the possibility of its connexion with other things.
>
> If I can think of an object in the context of an atomic fact, I cannot think of it apart from the *possibility* of this context. (TLP, 2.0121d–e)

Here, Wittgenstein illustrates that every object necessarily brings with it all its possibilities to occur in a specific context. While he mentions spatial and temporal objects in this remark, he will shortly after also speak of phenomenological objects such as colours and sounds. In this later remark the metaphor of "space" is used to illustrate a manifold of possibilities which, as it were, surrounds the objects:

> Every thing is, as it were, in a space of possible atomic facts. I can think of this space as empty, but not of the thing without the space. (TLP, 2.013)[59]

In this sense, "infinite space" is the space for spatial objects, but time, too, can be called a "space", namely a one-dimensional manifold for temporal objects: it is its necessary context apart from which it cannot be thought. In the same sense, there are colour space, pitch space and so on, as manifolds for colours, sounds and other types of objects (TLP, 2.0131).

These spaces are not *the* logical space, but are similar to the perceptual manifolds which were discussed in the last chapter. Wittgenstein speaks of such "Helmholtzian" manifolds like colour-space, pitch-space, space of hardness etc., i.e. perceptual or phenomenological manifolds, but also of a "spatial object in infinite space" by which he seems to mean physical or geometrical space.[60] These are all specific manifolds surrounding different kinds of

59. The Kantian ring (cf. CPR, A24/B38) of the second sentence has been pointed out to me by Shunichi Takagi, and it seems to confirm the intuition that this understanding of space as an *a priori* structure of possibilities takes up Kantian and Neo-Kantian traditions.

60. I understand it as a three-dimensional Euclidean coordinate system which is called "infinite", first, because it *is* infinite according to the axioms, but second, in order to avoid the awkward expression "spatial space" which baulks at the usage of "space" as a general metaphor for manifolds of possibilities. See also the note on Hertz's "unendlicher Raum" below.

objects, frameworks of possible atomic facts one of which the object must be part of: a coloured object can be red, a sound can be a C#, an object of the sense of touch can be hard, a spatial object can be at location (3, 5, 6) in a three-dimensional "infinite space", a temporal object at time *t* in a one-dimensional "time space", and so on. As in Helmholtz's manifolds, these objects lose their independence and become the "possibilities" of existence determined by various coordinates: this is what is meant by the remark that we cannot think of spatial objects apart from space and of temporal objects apart from time. A temporal object, for example, has to have a position in the one-dimensional time-manifold – it indeed *is* this position, again in the sense of Hyder's interpretation of Helmholtz's objects as "addresses in the point-fabric". This is the object's "form of dependence": it cannot exist apart from a space, apart from its occurrence in possible atomic facts. It is independent only insofar as it can occur in any of these possible atomic facts (TLP, 2.0122).

Logical space, now, is the most general of these manifolds. It is clear that the spaces listed in 2.0131 are all on a par, are different manifolds for different kinds of objects; and neither phenomenological (contra Hintikka and Hintikka 1986), nor physical manifolds (contra Graßhoff 1997) are preferred from a logical point of view: they are all manifolds of possibilities with a varying number of dimensions. These spaces can be combined in more complex *Sachlagen* as has already been shown in the section on Helmholtz: a blue, hard object at a certain position in "infinite space" can be represented in a five-dimensional manifold. Hence, a multi-dimensional all-encompassing space is thinkable which contains *all* possibilities, and this most general manifold is logical space.[61]

But back to objects: since they and their respective spaces of possibilities are necessary, they are the "fixed form" of the world, while their configuration, that is, their actual arrangement in facts, is changing and variable (TLP, 2.0271; 2.0272). What is necessary, doesn't change. Taking true and false propositions as a given and possibility as a pre-condition of truth,[62] Wittgenstein claims to have proven the "existence"[63] of objects. Objects and their internal properties determine which facts *can* obtain and this totality of possibilities, the "substance of the world" cannot itself be "merely possible" nor contingent. The so-called argument for substance (TLP, 2.0211; 2.0212) anticipates the notions of picture and proposition (which have not been introduced so far) and argues that

61. We will later encounter similar hierarchies of particular forms (space, time, colour) and the general "logical form", and particular pictures (spatial picture, coloured picture etc.) and the general "logical picture".

62. "In order for a proposition to be true it must first and foremost be capable of truth, and that is all that concerns logic." This remark is from the same day on which Wittgenstein introduces the notion of logical place: "for a statement to be possible the logical coordinates must really determine a logical place!" (NB, 29/10/1914).

63. Again, "existence" is not a good word (and Wittgenstein does not use it); what he says is that we have to assume objects in order to speak about facts. Sullivan highlights that the opposition of "fixed" and "changing" refers to *modal*, not *temporal*, variations (Sullivan 2001, 94). Hence, I prefer to call objects necessary and not "eternal" or "sempiternal" (e.g. Hacker 1986, 20).

(p1) we can form true or false pictures of the world, and

(p2) a proposition's sense does not depend on another proposition's truth, and hence

(c) the world has substance, namely the objects and their configurational possibilities.

The argument is notoriously brief and obscure, but it is clear that both "picture" and "proposition" in the *Tractatus* refer to the potentiality of facts and this potentiality is exactly what is guaranteed by Wittgenstein's objects.[64] For the internal properties of objects, their form, determine the "space of possible atomic facts" and these possible atomic facts are what pictures or elementary propositions correspond to.

The notion of propositions and their capacity to represent their sense independently of their truth is surely at the heart of Wittgenstein's book. His solution is implicitly prepared in the ontological considerations in propositions 1 and 2. Before we get to the picture theory, one more terminological issue needs to be clarified: the *structure* of an atomic fact is "the way in which the objects hang together", that is, it is a factive, contingent and external feature of objects which can hang together in an atomic fact or not. The *possibility* of this structure, however, is determined by the internal properties of the object and this is what is called *form*. Thus, form is the possibility that objects are arranged in a certain way in an actual fact – it is evident why this form is "fixed" as the possibilities do not change, only the actualities change. The "spaces" considered in 2.0131 are manifolds of possible atomic facts; the possibility of the structure of an atomic fact is its form. Therefore, these "spaces" are themselves kinds of forms: "Space, time and colour (colouredness) are forms of objects" (TLP, 2.0251). They are forms in the sense of possibilities of objects to be arranged in particular ways.

It has been said that the argument for objects is "transcendental" (cf. Hacker 2012a, 121); the objects can indeed be seen as the condition of the possibility of atomic facts: whether an atomic fact exists is contingent, but its possibility is guaranteed by the object's possibility to occur in this atomic fact, i.e. by its location in the "space of possible atomic facts" in which the object is necessarily situated (TLP, 2.013; 20131). If we extend Kant's epistemological system, first to manifolds in the 19th century sense and then to logic, and if we accept logical space as the most general manifold of possible atomic facts, we can indeed say that Wittgenstein's logical space, like Kant's space, is a "form", only not a form of intuition, but a logical form. Both are frameworks of possible empirical facts.

64. There are very detailed analyses of this argument (cf. Cheung 2017, 133–139); for my purpose the hint towards possibility may suffice: the necessity follows from the totality of possibilities.

If form and structure are distinguished along the lines possible-actual and necessary-contingent, then one is again tempted to reify form as a sort of "possible fact" with all awkwardness this expression implies. However, the possibility that objects are arranged in a certain way is not the same as the fact that objects are arranged in a certain way. In a retrospective comment on the *Tractatus* in 1931, when he is again thinking about possibilities, this time in the context of intentionality, Wittgenstein sees more clearly how confusing this mode of speaking is and suggests understanding facts simply as possibilities:

> The usage of the word 'fact' and 'deed' [*Tatsache* and *Tat*]. [...] It seems plausible to use the word 'deed' in such a way that it corresponds only to the *true* proposition. Hence, one would not speak of a deed that has not/never been committed. But the proposition 'this was a noble deed' must keep its sense even if I am wrong in assuming that what I call deed has happened. And this contains already all that is important and I can only make the stipulation to use the words 'deed', 'fact' (or also event) only in a proposition that completely affirms the obtaining of this fact. (MS110, 236–37)

> *[And a few remarks later:]* As to 'deed' and 'fact': it would be better to drop the constraint in the usage of these words as they only cause confusion, and to simply say: 'this deed has not been committed', 'this fact does not obtain', ' this event has not happened'. (MS110, 238)

Up to this point the focus of my study has been on the readings of possibility (1) and (2) and I have argued for an understanding of the factive concepts *Sachverhalt*, *Tatsache* and *Sachlage* as possible as opposed to necessary, but as actual as opposed to "merely possible". Objects and their internal properties, which determine their forms, are necessary as opposed to contingent, but cannot be actual in the sense of facts: they are categorically distinct from facts and "exist permanently", but this is a different mode of existence to the contingent existence of facts.[65] This ontology of possibilities is the basis of the picture theory which I now turn to.

3.1.2 Pictures and Propositions

The capability of propositions to represent facts independently of their existence is what the *Tractatus* aims to make understandable. Wittgenstein's solution is that propositions, if they

65. One could make a distinction between internal and external questions of existence similar to Carnap (1950, 22–23); "There are objects" has no sense as a statement of internal existence (cf. TLP, 4.1272). There are indeed some similarities between Carnap's conceptual frameworks and Wittgenstein's space concepts which shall become clearer when I discuss Wittgenstein's middle period in chapters 4 and 5.

have sense, describe not a fact, but the possibility of a fact. This solution is illustrated with two metaphors: possibility as a *picture* and possibility as a *location in logical space*. Both images are intertwined in the first comment on the picture theory:

> Wir machen uns Bilder der Tatsachen (TLP, 2.1)
>
> Das Bild stellt die Sachlage im logischen Raum, das Bestehen und Nichtbestehen von Sachverhalten vor. (TLP, 2.11)
>
> We make to ourselves pictures of facts.
>
> The picture presents the facts in logical space, the existence and non-existence of atomic facts.

I quote in German not in order to show any similarity to Hertz's *Bilder* – which is not as close as it seems –, but in order to highlight the language of possibility. As I also mentioned in the last chapter, Janik and Toulmin distinguish between *Darstellen* and *Vorstellen* along historical traditions: The first line is Hume–Mach–Vienna Circle where the concept of *Vorstellungen* is central: they are conceived as pictures in the mind which can be used to build a theory of "psycho-physics" or to construct the world logically out of sense data. In contrast, the line Kant–Hertz/Boltzmann–Wittgenstein deals with *Darstellungen*: it focuses on models, *Bilder*, of reality which do not claim to describe things in themselves, but only the schemes through which we know them. Unlike *Vorstellung*, this is essentially a public approach to the world; these systems, although not reaching to reality in itself, successfully capture aspects of reality. Instead *Vorstellungen* are basically private "images" in one's own mind, not accessible to others like the scientific "pictures" devised by the Neo-Kantians. The former also point to something outside of the system, sense-data, while the Kantian "spatial" approach determines boundaries *from inside* (cf. Janik and Toulmin 1973, 139–140).

This opposition is certainly interesting and illuminating, but I would like to add a further dimension to it which is based on my focus on possibilities: Another nuance between *Vorstellung* and *Darstellung*, not mentioned by Janik and Toulmin, is their modal difference. Although not capturing reality in itself, *Darstellung* implies the existence of what is represented; this is perhaps related to the underlying (partial, but nonetheless decided) realism of all these, roughly speaking, Kantian approaches. *Vorstellung*, in contrast, has the connotation of something that might not really exist; it is closer to imagination or presentation[66] and

66. See Schopenhauer's *Die Welt als Wille und Vorstellung*: the English title "The World as Will and Representation" was changed to "The World as Will and Presentation" in the latest translation by R. E. Aquila to capture this quasi theatrical meaning (2008, xiv). Schopenhauer indeed criticises the Kantian idea of things in themselves and considers the world as *Vorstellung* an illusion.

indeed emphasises the possibility rather than the actuality of what is presented. Janik and Toulmin do not ignore the occurrences of *vorstellen* in the TLP, but declare them negligible compared to the more important *darstellen*. However, the use of *vorstellen* can illuminate the difference between the two words and the role of possibility in the *Tractatus*. It confirms the "factive" nature of *Sachlage* which has to be supplemented with a marker of potentiality because Wittgenstein's logic is concerned with all possibilities. In the *Prototractatus*, one can see how Wittgenstein in 2.11 first wrote "das Bild [...] stellt dar" and later replaced "dar" with "vor" (MS104, 2.11). In a later remark commenting on what the picture has in common with what is pictured, Wittgenstein uses almost the same phrasing, but writes "dar" instead of "vor" and therefore has to add the adjective "möglich". In the final version, the striking parallelism of the two remarks highlights the difference in terminology:

> Das Bild stellt die Sachlage im logischen Raum, das Bestehen und Nichtbestehen von Sachverhalten vor. (TLP, 2.11)
>
> The picture presents the facts in logical space, the existence and non-existence of atomic facts.
>
> Das Bild stellt eine mögliche Sachlage im logischen Raume dar. (TLP, 2.202)
>
> The picture represents a possible state of affairs[67] in logical space.

The juxtaposition of these remarks supports the claim that it is important that what the picture *represents* is a *possible* situation since the situation could be different and the picture would still represent the same sense, whereas the adjective "possible" is not necessary when the situation is *presented* as this already implies the potential character of what is pictured.

To understand how pictures are meant to present facts, it is useful to follow the first level of comments on remark 2.1. Pictures are models and pictures "consist" of elements which correspond to objects in facts; "consist" has to be taken with a pinch of salt, just as atomic facts do not "consist" of objects: rather the picture consists in the combination of its elements in a definite way (TLP, 2.12–2.14). "That the elements of the picture are combined with one another in a definite way", Wittgenstein continues, "stellt vor" (!) "that the things are so combined with one another." Parallel to the terminology of *Sachverhalte*, this way of being combined is called the *structure* of the picture and the possibility of this structure is called *pictorial form* (TLP, 2.15). Now, a picture must have something in common with what is pictured (TLP, 2.16). Again we must not reify this "something": it is the *possibility* that the

67. It is interesting that the word *Sachlage* is translated differently although the context is very similar. This has probably to do with the problem to speak of "possible facts" in 2.202, although the "facts" in 2.11 are, of course, equally "possible" as they are "presented", not "represented".

picture's elements are arranged the same way as the objects are arranged in the fact whose picture it is: it is its pictorial form (TLP, 2.17). According to the various kinds of forms of objects we have encountered, there are various kinds of pictorial forms: there are spatial pictures, coloured pictures, temporal pictures etc. (TLP, 2.171). We have seen that all kinds of specific spaces can be represented in an all-encompassing logical space. Likewise "every picture is *also* a logical picture"; logical form is what every picture must have in common with reality (TLP, 2.18; 2.182). Every atomic fact, one could say, is in logical space, but not every atomic fact is in "infinite space" or "colour space". "The logical picture can depict the world" (TLP, 2.19), because it shares with the world the same most general form, they share the same possibilities, share the same logical space.

In the 3s, the modal status of pictures is, so to speak, "inherited" to thoughts because thoughts are logical pictures of facts. That the word "denkbar" plays an important role in the context of possibility and thinking may not be surprising, but the relation of Wittgenstein's usage of this word to Hertz's terminology is worth noting. In *Die Prinzipien der Mechanik* any position of a material point in infinite space[68] is called a *"denkbare Lage"* (Hertz 1894, 56). In the *Tractatus*, a picture represents a possible *Sachlage* in logical space and contains the possibility of the *Sachlage* which it represents (cf. TLP, 2.202; 2.203) – both its possibility aspect and its connection to the spatial framework are emphasised. In a parallel phrasing, Wittgenstein writes that the thought contains the possibility of the *Sachlage* of which it is a thought. The remark refers, again, to *Sachlagen* in logical space as possibilities and extends the Hertzian term "denkbar" from physical systems to the more general sphere of logic: "What is thinkable is also possible" (TLP, 3.02).

To conceive of atomic facts or situations as possibilities is repeatedly illustrated with spatial analogies: "The logical place and the geometrical place agree in that both are the possibility of an existence" (TLP, 3.411). Due to the delimiting character of Wittgenstein's enterprise this kind of possibility is particularly important as an antithesis to *impossibility*:

> We cannot think anything unlogical, for otherwise we should have to think unlogically. (TLP, 3.03)

> To present in language anything which "contradicts logic" is as impossible as in geometry to present by its co-ordinates a figure which contradicts the laws of space; or to give the co-ordinates of a point which does not exist. (TLP, 3.032)

68. Like Wittgenstein in TLP 2.0131, Hertz speaks of objects "im unendlichen Raume". I take it that both mean something like a three-dimensional Euclidean coordinate system which lends itself to geometrical or physical application. The difference is that Hertz is interested *only* in this space and he uses it as a means to model physical systems; whereas Wittgenstein speaks of "infinite space" as one of many possible manifolds, the most general of which is logical space. Therefore, his concept of "denkbar" includes *everything* thinkable, while Hertz's "denkbar" is a shorthand for "geometrically conceivable" (Hertz 1899, 48).

> We could present spatially an atomic fact which contradicted the laws of physics, but not one which contradicted the laws of geometry. (TLP, 3.0321)

This delimiting function of the determination of possibilities can perhaps be called the Kantian aspect in both Hertz's and Wittgenstein's systems. In Kant, the forms of intuition, space and time, determine (together with the categories) the possibilities of empirical reality as it can be known by pure reason, thereby delimiting its sphere. In Hertz and Wittgenstein, the possibilities are likewise determined *a priori*, by Hertz in the field of mechanics, by Wittgenstein in logic. The latter's aim is to delimit "thinking, or rather: the expression of thought" and this limit is further specified as a limit that distinguishes language from "nonsense" (TLP, preface).[69]

Now words in the preface are not always used in the sense in which they are used within the book,[70] but if "nonsense" is roughly related to what is called "sense" within the system of the *Tractatus*, then it is not so difficult to understand: sense is "what a picture represents" (TLP, 2.221) and we have already seen the remark clarifying what the picture represents: it is a possible situation in logical space (TLP, 2.202). Hence, the sense of a picture is a location, a possibility in logical space and here "possibility" can be read in the third sense: as opposed to impossibility. Nonsense is what is *not* localisable in logical space, what cannot be pictured by means of a logical picture, that is, by a thought. Since the thought is "the proposition with a sense"(TLP 1961, 4),[71] language, too, is thus bounded by this realm of possibilities which is exhausted by logic.

> Logic fills the world: the limits of the world are also its limits.
>
> We cannot therefore say in logic: This and this there is in the world, that there is not.
>
> For that would apparently presuppose that we exclude certain possibilities, and this cannot be the case since otherwise logic must get outside the limits of the world: that is, if it could consider these limits from the other side also.
>
> What we cannot think, that we cannot think: we cannot therefore say what we cannot think. (TLP, 5.61)

69. It is a limit in the sense of the Moore-Sullivan dispute mentioned in chapter 2: we cannot know both sides of the boundary.

70. It says that in the book "thoughts" are expressed although the propositions of the *Tractatus* are not thoughts in the sense of "logical pictures of facts"; their truth is allegedly "unassailable" (*unantastbar*) while the system says precisely that any truth is contingent and therefore "*antastbar*" (disputable).

71. Here, the Pears/McGuinness translation has the advantage of keeping the crucial word sense which is lost in Ogden's translation of "*sinnvoll*" as "significant".

Since logic speaks of all possibilities, possibility is not a property a fact could have or not. This is what is meant with the famous expression "delimiting from within": anything outside of logical space we cannot even speak of. Propositions without sense are not "possible" in the three senses of the word I have distinguished: they are impossible in the sense of (3) and can therefore not even be assessed with regards to criterion (1), whether they are correct or not, refer to an actual fact or not. With regards to (2) there are two special cases of propositions that do not have sense. Since "sense" was defined as the possibility that objects can be combined in a certain way, a place in logical space, contingency seems to be a condition for "sense". For objects can be combined in a certain way or not and the analogy of a "place in logical space" was precisely used to highlight this aspect of possibility. Tautologies and contradictions are not "possible" in this sense: for their truth is certain or impossible, but not contingent. Hence, they do not have a "sense" in the Tractarian usage of the word, they cannot be pictured and they do not have a specific location in logical space. However, these special propositions are still related with that space:

> Tautology leaves to reality the whole infinite logical space; contradiction fills the whole logical space and leaves no point to reality. Neither of them, therefore, can in any way determine reality. (TLP, 4.463)

> The truth of tautology is certain, of propositions possible, of contradiction impossible. (TLP, 4.464)

This kind of being senseless (*sinnlos*) is explicitly distinguished from nonsense (*Unsinn*) which is completely outside of logical space; it is, to speak with a metaphor by Elisabeth Anscombe, not even "on the map" – a metaphor with which she aims to illustrate the connection between logical space and negation.[72]

3.2 Logical Constants and Logical Space

One of Wittgenstein's main objectives is to explain how we can form true or false propositions about the world; *that* this is possible is taken as a given, it is the starting point of the *Tractatus*. Since it is essential for a proposition that it can be true or false (cf. Bonino 2008, 53–59), negation must, in a way, leave the sense of the proposition untouched. Attempts to meet this requirement pervade the pre-Tractarian notebooks. The issue of negation clearly becomes acute after Wittgenstein has come up with both metaphors for the potentiality of propositions:

72. She illustrates the location of a *Sachlage* in logical space by means of an island on a map whose opposite would be the rest of the map. Not without reason she talks about logical space in the lecture on negation (cf. Anscombe 1959, 75).

picture and space. It turns out that they do not account for the problem equally well. If a picture is true it corresponds to a fact; propositions can say that something is *not* the case and if this "something", a "possible fact", does indeed not exist, then the "negative proposition" is true. But also this true negative proposition must be a picture and how can a picture show what is not the case? If we consider physical pictures, drawings, paintings, one could think of a crossed-out or inverted picture, but none of these solutions is satisfying, and in any case the negative picture cannot show what is not the case without its positive counterpart:

> If a picture represents what-is-not-the-case in the forementioned way, this only happens through its presenting *that* which *is* not the case.
>
> For the picture says, as it were: "*This* is how it is *not*", and to the question "*How* is it not?" just the positive proposition is the answer. (NB, 3/11/1914)

This brings us back to the problem of the sense of a picture which, we learned, is a possible situation in logical space. Let us again follow an argumentative path in the decimal system to better understand this point. We have seen that "the thought is the proposition with a sense" (TLP, 4) from which it follows that propositions with sense, like thoughts (cf. TLP, 3), are logical pictures of facts. In the sub-remarks to this main proposition (the 4.0s), Wittgenstein is concerned with the nature of this pictoriality:

> The proposition is a picture of reality.
>
> The proposition is a model of the reality as we think it is. (TLP, 4.01)
>
> This we see from the fact that we understand the sense of the propositional sign, without having had it explained to us. (TLP, 4.02)

A proposition has sense by describing a possible *Sachlage* in logical space, and it must therefore be connected to this *Sachlage*. This connection is precisely that it is its logical picture.

> The proposition only asserts something, in so far as it is a picture. (TLP, 4.03)

Pictoriality has already been explained in the 2s as, first, a *representation* of objects by elements in the picture and, second, as an *identity* of form, that is: the elements in the picture can be arranged in the same way as the objects, for which they stand, can be arranged in a fact. The *Grundgedanke*, which has received much attention in the literature, is in fact a comment on this remark 4.03 which concerns the representation of objects in pictures:

> The possibility of propositions is based upon the principle of the representation of objects by signs.
>
> My fundamental thought is that the "logical constants" do not represent. That the *logic* of the facts cannot be represented. (TLP, 4.0312)[73]

The notion of representation is hence tied to pictoriality. That the logic of facts cannot be represented is related to the claims that the picture cannot represent its form which is after all a logical term. Logic is concerned with possibilities, whereas the picture can indeed present a possibility, but only, strange to say, as if it were actual because it cannot represent what is not the case without representing the non-obtaining state of affairs. Hence, the "logical picture can depict the world" (TLP, 2.19), where the world is understood as the totality of facts, but it cannot represent all possible worlds. The totality of possibilities, which Wittgenstein wants to capture in logic, cannot be represented in the picture theory. Therefore, the logical constants, which determine all configurational possibilities of objects, do not represent, the logic of facts is not representable.

Proposition 4.04 elaborates the idea of representation: pictoriality is a one-to-one mapping between proposition and *Sachlage* and if things change in reality, its representatives in the picture change accordingly (Wittgenstein draws an analogy to Hertz's dynamical models). Proposition 4.05 concerns the comparison of a picture with reality which was already present in the 2s ("It is like a scale applied to reality", TLP 2.1512), but is now extended to propositions: the proposition is compared with reality. Now this comparison brings in, again, the bipolarity of propositions; for the comparison with reality will yield a result: the proposition is true or false. The next step in the 4.0x series expounds this with regards to negation:

> Propositions can be true or false only by being pictures of the reality. (TLP, 4.06)

The proposition has a sense "independent of the facts"; $\sim p$ could by convention refer to the *Sachlage* we now refer to with p. But this would not mean that $\sim p$ somehow refers wrongly

73. There is a tension between the "fundamental" word *Grundgedanke* and its low number which can be interpreted in different ways: Schulte suggests that the numbering system does not actually indicate a hierarchy since the "fundamental thought" has such a low number (Schulte 2005, 63). In contrast, Bonino argues that the fundamental thought is in fact not to be understood as fundamental in the sense that all other propositions can be derived from it (Bonino 2008, 125). I tend to the latter view as the remark itself is in fact implied in other remarks and can indeed be said to be itself *based* on the picture theory. It does not found the rest of the system. The formal equivalence of fact, picture, thought and proposition by means of picturing relations, which is spelled out in the higher numbered remarks, has in a profound sense more "logical weight" than the pronouncement of its motivations or implications. The relative prominence of the *Grundgedanke* in the literature may be related with the fact that it can be shown to be a reply to Frege's and Russell's theories about logical constants (e.g. Bonino 2008, 125–129) and thereby can be presented as the solution to a well-defined problem – a welcome rarity in Wittgenstein scholarship.

to the right fact – it *is* the true proposition, it means what we normally mean by p; in that notation the absence of "~" would be used like "~" is used now (TLP, 4.061; 4.062).

> That, however, the signs "p" and "~~p" *can* say the same thing is important, for it shows that the sign "~" corresponds to nothing in reality.
>
> That negation occurs in a proposition, is no characteristic of its sense (~~ $p = p$).
>
> The propositions "p" and "~p" have opposite senses, but to them corresponds one and the same reality. (TLP, 4.0621)

This last remark is obviously puzzling: how can the senses be opposed to one another, but refer to the same reality? The same problem is expressed in the continuation of the above quoted passage in the wartime manuscripts where Wittgenstein within a few lines seems to straightforwardly contradict himself; however, the solution is also indicated in this passage by moving from the picture-imagery towards space-imagery which is better suited to account for the special role of negation with regards to sense:

> It might be said: The negation refers to the very logical place which is determined by the negated proposition. [...]
>
> The negating proposition refers to a *different* logical place from the negated proposition. (NB, 3/11/1914)

We have seen how opposite pictures correspond to the same reality insofar as a negative picture can only represent how it is not, by relating to its positive counterpart. To understand how the senses are nevertheless opposed to one another, it is helpful to distinguish between a "picture sense" and an "arrow sense"[74] (Pilch 2017, 19) which accounts for the essential bipolarity of propositions. The picture sense is shared by proposition and negation and is *shown*, while their arrow senses are opposed to one another and *said*:

> The proposition *shows how* things stand, if it is true. And it *says*, *that* they do so stand (TLP, 4.022).

Note how the metaphors are crossed here: the showing aspect clearly belongs to the picture-side and its exhibition of form without saying it; the arrow aspect, the direction, the affirma-

74. This idea takes up Wittgenstein's metaphor: "propositions resemble arrows: they have sense" (TLP, 3.144). Wittgenstein does not actually make the distinction, his use of the word "sense" rather seems to shift between them or include both aspects. For methodological purposes, the distinction is useful though. Note also that the German word for sense, "*Sinn*", is etymologically related to "direction" and hence implies something arrow-like, something vectorial (cf. Grimm and Grimm 1854, Vol. 16, Sp. 1103, *Sinn*, I.3: "*die ursprüngliche bedeutung der wurzel war augenscheinlich die einer ortsbewegung*").

tion and negation, are better captured in spatial imagery.[75] In logical space, the idea that a proposition and its negation are related, but distinct, can be illustrated more intuitively and this is indeed what Wittgenstein does in the seemingly contradictory manuscript entry which is later integrated into the final version of the *Tractatus*:

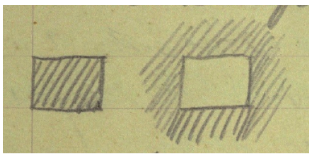
> Every proposition must *already* have a sense; assertion cannot give it a sense, for what it asserts is the sense itself. And the same holds of denial, etc. (TLP, 4.064)
>
> One could say, the denial is already related to the logical place determined by the proposition that is denied.
>
> The denying proposition determines a logical place other than does the proposition denied.
>
> The denying proposition determines a logical place, with the help of the logical place of the proposition denied, by saying that it lies outside the latter place. (TLP, 4.0641)

It seems to be paradoxical, but is in fact the solution. The picture sense, as it were, describes the boundary of a proposition which it shares with its negation. Thereby the negative proposition is related to the same logical place. It determines a *different* logical place by covering everything *outside* this boundary, while the "positive proposition" covers only the area *inside* the boundary. Negation is an operation. It consists in switching the side of the boundary, is therefore not related to the pictorial, representing aspect of Wittgenstein's system, but to the spatial, formal aspect. It is important that here logical place is not conceived as a point, but rather as an extended area in logical space.[76] At one point Wittgenstein conceived of this internal relation as a spatial boundary in a quite literal, physical sense: an object blocking an area, so that it is clear: you *cannot* go there, this area is excluded from your possibilities.

> Think of the representation of *negative* facts by means of models. E.g.: two railway trains must not stand on the rails in such-and-such a way. The proposition, the picture, the model, are – in the negative sense – like a solid body restricting the freedom of movement of others; in the positive sense, like the space bounded by solid substance, in which there is room for a body.

75. The simile in 4.063 highlights that this "arrow" sense belongs to the proposition intrinsically. Therefore, being true or false cannot just be represented by a point on a paper being black or white – these properties are external to the point and one could point to it without even knowing what black is. Rather, the true and false propositions must be different areas in logical space which are complementary to one another.

76. This is meant by Anscombe's metaphor of the island on a map whose negation would be the rest of the map (1959, 75). The role of space in Wittgenstein's treatment of negation is also highlighted by Pilch who conceives of a proposition and its negation as regions in logical space, too (2017, 19).



This image is very clear and *must* lead to the solution (NB, 14/11/1914)

This imagery of space is also better than that of pictures to explain the special role of tautologies and contradictions. For what should a picture of "It is raining or it is not raining" look like? Indeed, the comparison between negative facts and physical bodies is set in the context of Wittgenstein's discussion of tautology and contradiction which was quoted above: contradiction, as it were, blocks the whole space and leaves nothing to the proposition, tautology leaves everything unblocked so that the proposition is true no matter what (TLP, 4.463). In Anscombe's image, the whole map would consist of island so that any point can be said to be "on the island"; or the whole map is ocean so that no point can be on the island.

The transfer of this idea to logic can be illustrated by means of the spatial imagery: a proposition and its negation are two complementary regions in logical space. Crucially, in this space there is a region representing p and a region representing $\sim p$ – but there is no point or region representing the symbol of negation itself. Negating something just consists in switching the side of the boundary.

What about other logical constants than negation? Pilch has shown that also other operations, such as implication, can be interpreted spatially if the functions of logical space are conceived in different "representation spaces". This interpretation of logical space is based on the TF-notation which is itself designed to do away with logical constants in the sense of logical objects. This notation makes use of the possibility to represent propositions by means of truth-tables. Two elementary propositions and their disjunction can thus be represented as follows:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Taking only the last column of tables like this, every logical combination of the two elementary propositions can be represented with a unique sequence of Ts and Fs. The truth-conditions, as Wittgenstein calls them, of the disjunction of two elementary propositions p and q are TTTF; the truth-conditions for conjunction would be TFFF; a mere affirmation

of p: TTFF, and its negation: FFTT. The notation consists of the truth-conditions and a declaration of the elementary propositions that are involved, e.g. (TTTF)(p, q). A scheme of all permutations of two elementary propositions is given in TLP 5.101. With this notation all internal relations between propositions can be expressed without the need for special signs for negation, disjunction or implication. Logical operations simply are transitions from certain distributions of Ts and Fs to other distributions. Negation for example turns all Fs into Ts and vice versa. And it is clear why with this method double negation yields the same result as affirmation.

The idea that logical constants are superfluous is discussed in more detail in proposition 5.4. After having introduced the mechanism of truth-operations applied to truth-functions (i.e. co-ordinations of truth-value to elementary propositions), he states:

> All propositions are results of truth-operations on the elementary propositions.
>
> The truth-operation is the way in which a truth-function arises from elementary propositions.
>
> According to the nature of truth-operations, in the same way as out of elementary propositions arise their truth-functions, from truth-functions arises a new one. Every truth-operation creates from truth-functions of elementary propositions, another truth-function of elementary propositions i.e. a proposition. The result of every truth-operation on the results of truth-operations on elementary propositions is also the result of *one* truth-operation on elementary propositions.
>
> Every proposition is the result of truth-operations on elementary propositions. (TLP, 5.3)

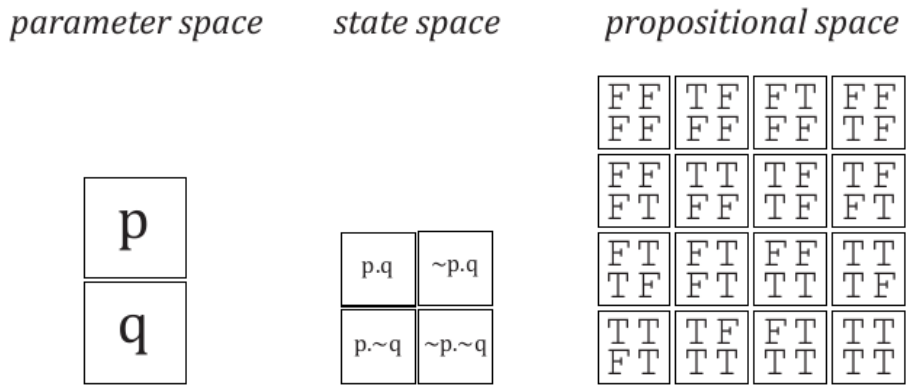
> Here it becomes clear that there are no such things as "logical objects" or "logical constants" (in the sense of Frege and Russell). (TLP, 5.4)

> For all those results of truth-operations on truth-functions are identical, which are one and the same truth-function of elementary propositions. (TLP, 5.41)

This argument seems to me to be related to the spatial account of logic that has been sketched. Many truth-operations can result in the same distribution of Ts and Fs, the same truth-conditions, that is, in the same "area" in logical space, the same truth-function of elementary propositions: the operations do not have a fixed location in that space, they are relations, transitions, between regions.

What is important is the role of space in this central thought of the *Tractatus*. Wittgenstein's idea can be represented in three representation spaces that together contain all functions

of the abstract structure "logical space"; all mappings of elementary propositions with a truth-value and all operations on them can be represented with this structure, as Pilch has pointed out. In what he calls "parameter space", the fundamental bipolarity of elementary propositions is captured. This space has as many "points" as there are elementary propositions.[77] For n elementary propositions there are 2^n possible conjunctions (Wittgenstein's variable K_n, TLP 4.27) which can be represented as states in "state space". This space only contains the conjunctions of elementary propositions, that is, for two elementary propositions:, there are four possibilities of truth and falsity: $p.q$, $\sim p.q$, $p.\sim q$ and $\sim p.\sim q$.[78] Applying the other truth-operations to these states leads us to the totality of combinatorial possibilities of two elementary propositions. There are 2^{2^n} possibilities here which corresponds to Wittgenstein's variable L_n (TLP, 4.42). It stands for the number of possibilities for propositions to be true or false. Since this exhausts all propositions that can be constructed out of the given elementary propositions, Pilch calls this space "propositional space".[79]



Representation Spaces after Pilch 2017, 39

This shows that the idea of logical space in its various features and functions serves as a means to illustrate Wittgenstein's fundamental thought. It shows how the possibilities of propositions as understood in the *Tractatus*, as the transformations of truth-functions of elementary propositions by means of truth-operations, can be represented without the

77. The number of elementary propositions cannot be known, but Wittgenstein shows how he imagines this space by using toy-spaces of up to three elementary propositions.

78. Note that this is what corresponds to *possible* facts according to Wittgenstein's explanation to Russell: "Tatsache is what corresponds to the logical product of elementary propositions if it is true".

79. Sullivan is on the same track when he distinguishes two models of logical space which should not be considered as alternative, but additional to one another. The first model is a space of as many dimensions as there are *Sachverhalte* each of which can only exist or not ($\approx$ parameter space). The second model is very similar to Pilch's state space, only that Sullivan tries to explain the features of propositional space within the same framework – which is possible, but Pilch's visualisation is more perspicuous (cf. Sullivan 2001, 96–97).

need for logical constants, without reification of logical operators. These "do not represent" because they are no locations in logical space which pictures refer to if they represent. The spatial account of logic can illustrate the logical operations by internal structures as Pilch demonstrates for negation and logical consequence (cf. Pilch 2017, 41–45). The metaphor of "picture" alone could explain possibility, too, but it cannot account for negation because the "picture sense" does only show *how* things *can* be arranged, whereas the "arrow sense", which is best illustrated spatially, says *that* they are so arranged (cf. TLP, 4.022). This explains also the apparent paradox that the negation is related to the same logical place as the negated proposition, but determines a *different* logical place: they share the same boundary in space, but the positive proposition affirms everything inside the boundary while the negative proposition affirms everything outside the boundary (TLP, 4.0641).

It is worth noting how important the problem of negation is in the development and conception of this solution. This is a constant in Wittgenstein's thinking who again and again uses the method of asking: what would be the opposite? Is there an antithesis at all? It is a consequence of his conceiving of sense-frameworks by means of spaces which essentially feature this divisibility into complementary parts. This remains vital in the so-called middle period which I will present in the next chapter, for example when discussing grammatical, or as he sometimes calls them: geometrical, propositions in the *Big Typescript*: "What does it mean when one says: 'I can't imagine the opposite of that' or 'What would it be like if it were otherwise?'?" (MS213, 95r).[80] A similar emphasis is made in the *Blue Book* where our attention is directed to what Wittgenstein calls the metaphysical use, "namely without an antithesis" (BB, 46); and of course the same point is made in the private language argument and its discussions of the privacy of pain: Can I know that I am in pain if it is unthinkable that I am in pain and don't know it? – These statements which seem to have no alternative, i.e. necessary statements, are always suspicious for Wittgenstein; and mostly they turn out to be what he calls grammatical or indeed *geometrical* propositions (e.g. MS105, 43; BT 306r, 749r; MS119, 61–62; MS122, 69v). Remarks that simply state the structure of the space, but do not tell us anything new, are not empirical insights. Nor are they metaphysical insights, the recognition of a metaphysical *truth*, for at least the later Wittgenstein would understand geometry and grammar in a (qualified) conventionalist sense. Grammatical propositions, which simply describe how a conceptual space works, do not have sense according to the criteria they themselves determine. A clock defines a space of meaningful time expressions, but the clock itself is not a time expression and is thereby nonsense within the space of time expressions. The *Tractatus* can be compared to the description of such a clock, with the peculiarity that the clock is supposed to determine *everything* sayable so that it itself

80. Translation by Luckhardt and Aue (BT 2005, 78e).

cannot be described at all: while the clock can be described with non-time-expressions, logical space could only be described with non-logical propositions. Hence, the building and describing of the clock itself is nonsense in a "resolute" sense, but it is nonsense due to the clock being as it is. Does it at least show the "real time"? Is the world really as described in the *Tractatus*? Wittgenstein famously described the book later as "a clock that didn't work" which is often quoted by standard readers to show that it is not complete nonsense in the "resolute" sense (cf. Hacker 2017, 217). However, the clock can also be read as a simile of a measure: of determining criteria for something which can be true or false, as a space which constitutes sense. Whether the clock itself shows the right time one cannot really know, because it sets the standard of what "the right time" is. Therefore, one could indeed say its truth is "unassailable and definitive" insofar as it is a *definition*; but it is not true (nor false) in the sense of a proposition within the space: it is nonsense. While I shall argue that this grammatically conventionalist view is a characteristic of the later Wittgenstein, in the *Tractatus* matters are not so clear. For there indeed seems to be only *one* logical space, only *one* grammar, only *one* geometry. It is not quite clear why this must be so, but it seems that Wittgenstein did hold this view in his first book.[81] This shall be the topic of the next subsection.

3.3 Universality and Uniformity – the Structure of the World

It has been noticed that a main motivation for the system of the *Tractatus* was Wittgenstein's rejection of a "universalist conception of logic" which he found in Frege and Russell (Ricketts 2018, 54). It is the view that logic has the same status as a science and is distinguished merely by being the most general science; that logic encloses all the other sciences by investigating the most general truths about the world. Although rejecting *this* kind of universalism, Wittgenstein still

> adheres to the idea of a single framework embracing every sentence (see 6.124). There is just one logical space in which every sentence with sense determines a location. Each sentence with sense is related to every other, if only by the relation of independence (Ricketts 2018, 80)

81. Denis McManus reconstructs the argument leading to a "vertically unified" logic as opposed to Russell's theory of a hierarchy of logical types (cf. 2009). What is especially important here is the demand that logic should be completely foreseeable (cf. Sullivan 2002, 58). If there were types, new kinds of propositions could be discovered for which specific rules would apply; this question of numbers and kinds occurs in the retrospective treatment of that issue by Wittgenstein which I present in the next section. However, I will not give a detailed account of Wittgenstein's rejection of type theory for there is not much in Wittgenstein's own text about this issue; it has been extensively discussed by others whom McManus quotes (Potter, Sullivan).

In light of its status as the most general manifold, it is clear why logical space always occurs in a universal singular as *the* logical space (TLP, 1.13; 2.11; 2.202; 3.4 ff.; 4.4623). The completeness of logical space is a requirement for the whole system. It guarantees that every meaningful proposition is "localizable and by this localization linked to a possible Sachverhalt, and on the other hand it clarifies negation to be a relation between complementary regions in logical space" (Pilch 2017, 18). The mechanism of operations, negation, disjunction etc., only works if the space is complete. Negation leaves open *all* other possibilities, its region in logical space is really complementary to the negated proposition: each elementary proposition divides logical space into two parts. "If all the positive statements about a thing are made, aren't all the negative ones already made too? And that is the whole point." (NB, 26/11/1914). This is again related with the avoidance of logical constants. If we remember the TF-notation and its relation to space we can see how conjunction and disjunction, "logical product" and "logical sum", presuppose the complete space in a similar way. If only two elementary propositions are given, each place, each proposition in the propositional space of 16 propositions, contains the possibility of the other places, i.e. of the other propositions, by allowing for variations of its distribution of Ts and Fs. Wittgenstein highlights this completion function in a comment on section 3.4 which is dedicated to logical space:

> Although a proposition may only determine one place in logical space, the whole logical space must already be given by it.
>
> (Otherwise denial, the logical sum, the logical product, etc., would always introduce new elements—in co-ordination.)
>
> (The logical scaffolding round the picture determines the logical space. The proposition reaches through the whole logical space) (TLP, 3.42)

The whole issue of completeness is very important also for the later Wittgenstein. As we will see, he will not argue for the generality and uniqueness of space anymore, but he will always insist that each conceptual space is complete.[82] He argues that the mathematical space of rational numbers has no "gaps" which had to be filled with irrational numbers (MS111, 29), that the "vagueness" of the visual field cannot be captured in a greater space, for instance by an "exact account of inexactitude" (cf. MS107, 172);[83] in the *Investigations*, he asks us to conceive of the simple language-game of the builders as a "complete primitive language"

82. Rhees warns against equating completeness with unity (1970, 51–52). He takes Wittgenstein to be directed against completeness, not unity, when he argues against the "craving for generality" in the *Blue Book* (18–19). In my view, however, generality is again different from these two notions. A space, like colour space, can be complete without being general.

83. And this completeness is related to the grammatical fact, that "vagueness" has no opposite in visual space.

(PI, §2); games can be games – complete games – even if not *everything* is determined by the rules. And introducing new rules is then essentially inventing a new game. Treating simple geometries as precursors of one great unifying geometry is repeatedly characterised as "unjust" or "unfair", mostly with the shorthand "Nicod, Russell" (MS213, 202; MS115, 81; MS142, 19) referring to Nicod's *Foundations of Geometry & Induction* and Russell's preface in which he describes just this method (cf. Nicod 1930). In the *Investigations*, Wittgenstein elaborates his account of completeness without universality by admitting that Augustine's language is indeed a system – and a complete system as has been highlighted in §2 – "only not everything that we call language is this system."

> And one has to say this in several cases where the question arises "Will that description do or not?" The answer is: "Yes, it will, but only for this narrowly circumscribed area, not for the whole of what you were purporting to describe. (PI, §3)

This is a powerful criticism of his earlier view right at the start of his envisaged second book which he had planned to publish together with the *Tractatus* as he remarks in the preface from 1945 and in a tentative title which is considered in a manuscript: "Philos. Untersuchungen der Log. Phil. Abh. entgegengestellt" (MS128, 52).

Generally, the *Tractatus* uses a sort of generalising language: this is already evident from its bold opening statement about "the world"; and also logical space appears to be universal and uniform. Later commentators have ascribed a certain "logical absolutism" to the *Tractatus*[84] and Wittgenstein himself takes this as one of the major points of revision on his way to his later philosophy: he does so explicitly in an argument from 1937 which shall be treated below, and implicitly, performatively, in the development of a plurality of grammatical spaces with their own context-dependent logics in the writings after 1929 which are not treated as sub-spaces of an all-encompassing logical space.

Another symptom of the striving for universality in the *Tractatus* is the search for the general form of the proposition which follows from the generality and completeness of logical space (which is itself a kind of "form" as we have seen). In the remarks on the non-existence of logical constants (5.4), there are some arguments that emphasise this unity and shall later be recognised as problematic. From 5.453 ("there are no numbers in logic"), Wittgenstein claims the neatness, or rather: simplicity (*Einfachheit*) and apriority of logic: there are not many, but only *one* logical constant which he finally calls "the general form of the proposition".

84. As opposed to Carnap's "relativized" view on logic as expressed in his *Principle of Tolerance* (Friedman 1999, 183). In fact, however, Wittgenstein made a similar move as Carnap with regards to "logical absolutism" in his philosophy after 1929.

> It is clear that everything which can be said *beforehand* about the form of *all* propositions at all can be said *on one occasion.*
>
> For all logical operations are already contained in the elementary proposition. For "$f\,a$"[85] says the same as "$(\exists x).fx.x = a$".
>
> Where there is composition, there is argument and function, and where these are, all logical constants already are.
>
> One could say: the one logical constant is that which all propositions, according to their nature, have in common with one another.
>
> That however is the general form of proposition. (TLP, 5.47)

In the sub-remarks to 5.47, this gets further sublimated as "the essence of proposition" and equated with "the essence of the world" (TLP, 5.471; 5.4711). In 1937 when Wittgenstein drafts the first version of PI §§89–116, he criticises these remarks in particular and comes to realise that the assumed unity and "crystalline purity" of logic (and world) were a postulate, and not a truth about the world (cf. MS157a, 45 ff.).

In this notebook, Wittgenstein takes up various ideas from the *Tractatus* and tries to find out why he was looking for a pure and unique essence of language in the first place. The reconstruction has five steps:

1. He was concerned not with facts, but with language – and this was correct. For language can also describe those facts that do not exist.

2. Since he was concerned with language, he naturally asked for the *essence* of language.

3. He wanted to get to the essence of language *once and for all*, it had to be independent of experience and *a priori*. For this reason there could be no *kinds* of language or parts thereof – for if there was a certain number of kinds, it would already be empirical and new kinds could possibly be discovered (MS157, 47r). Hence: „All numbers in logic must be capable of justification. Or rather it must become plain that there are no numbers in logic" (TLP, 5.453).

4. Hence, the answer to the question for the essence of language must be one-fold or simple (*einfach*). "And so it seemed as if our answer had to be *simple* to the highest

85. This function-argument variable is, by the way, the closest we get to an example of an elementary proposition. It contains everything that is said by the second expression which uses quantifier, conjunction and identity. "A point of space is an argument place", Wittgenstein says in the section on objects and their dependence on the facts in which they can occur (TLP, 2.0131): I take it that an object can be understood as the "*a*" in the elementary proposition "*fa*"; already here the metaphor of space is connected to the later ideas of logical form and the non-existence of logical constants.

degree. Yes, actually not even *one*-fold" (MS157a, 47r). Logic had to be *one* in this metaphysical sense (without alternative), and also "proposition" and "thought" had to be *one,* and finally there was an equivalent series: "world, thinking, language, proposition. Thus the question for the essence of proposition (for the general form of the proposition) was the question for the essence of the world" (MS157a, 47v–48r).

5. This claim is the "sublime" against which Wittgenstein will argue in the philosophy chapter of the *Investigations* – and this claim cannot be reduced, or „scaled down". This preconception which is intimately connected with that of the "crystalline purity of logic" can only be removed "by turning our whole inquiry around" (PI, §108).

In the whole *Nachlass*, this is the most explicit discussion of Wittgenstein's view on the unity of logic and the world in the *Tractatus*. They are presented in the notebooks MS157a and 157b. Their importance for him is shown by the passionate tone in which they are written (it resembles the tone from the wartime notebooks); moreover, they make up the core of the philosophy chapter in the *Investigations*, the most carefully crafted piece of the entire *Nachlass*. The argument has again a connection to the Kantian ideas presented in the last chapter, although it does not correspond to the Kantian argument for the transcendental unity of apperception. But it is connected to the Kantian notion of *a priori*: precisely because logic is independent of experience it has to be *one*-fold in the strong sense that it strictly speaking does not even allow for numbers. In the notebook, he quotes the first sentence of TLP 5.453, "All numbers in logic must be capable of justification" (MS157a, 68r), which is followed in the *Tractatus* by: "Or rather it must become plain that there are no numbers in logic". The remarkable parallel between the development of geometry after Kant and Wittgenstein's usage of "space" is that both Kant and Wittgenstein took one *a priori* form as the only possible one, and that only later different forms, different spaces were recognised as possible. Unlike Kant, Wittgenstein was able to do this correction himself. "The a priori must become *one* form of representation. i.e. this concept must be stripped of its nimbus" (MS157b, 3v).

3.4 Summary

In this chapter I investigated essential characteristics and functions of Wittgenstein's usage of space metaphors in the *Tractatus*. First, I showed how logical space, together with the picture theory, illustrates the ontology of possibilities which underlies the capability of language to express sense independently of truth. I analysed the interdependence of facts, objects, and form as devised in the beginning of the book and provided an interpretation of pictures,

thoughts, and propositions in light of this modal ontology. With regards to propositions the notion of negation became central as it bundles the problem of possibility and representation: How are the negative and the positive picture related which they surely must be? For if one is false, the other is true and vice versa. It turns out that the spatial imagery is better suited to meet this requirement than the metaphor of pictures. Indeed, the idea of logical space underlies the whole mechanism of logical operations which can be conceived as transitions in a space of possible *Sachlagen*. This also provides an explanation of "nonsense" in the *Tractatus*: it is what is not localisable in this space and can therefore in a straightforward sense not be spoken of. Something ineffable can be shown, by showing how the space works; but something that is not even related to logical space is "simply nonsense", as Wittgenstein puts it in the preface. Finally, I explained the alleged unity of the *Tractatus* which manifests itself in its craving for generality, its attempt to think of logical space as the one and only super-structure of all possibilities.

One similarity between Kant's (Euclidean) space and Wittgenstein's logical space is that both are supposed to be *a priori* and apodictic, i.e. the only conceivable *form* of reality (in Kant's "aesthetical" account: the empirical world as perceived through the "outer sense"; in Wittgenstein's logical account: the totality of facts). Wittgenstein generalises the idea of an *a priori* form of knowledge from a merely "spatial space" to an all-encompassing logical space. Logic is for him an investigation of the conditions of knowledge, it is "transcendental" (TLP, 6.13). However, what happened to Kant's *a priori* form of intuition, Euclidean space, also happened to Wittgenstein's logical space. He has to acknowledge that different *a priori* "spaces" are conceivable and that there is an element of experience in their determination.

When Wittgenstein starts working on his later philosophy, he performs a shift from unity to plurality, from one all-encompassing logical space to many grammatical spaces. This shift shall be investigated in the next chapter. The features of logical space, which I discussed in this chapter, shall, however, be important in all variations of the idea of spatial representation: it is always a framework of *possibilities* which are determined by a "geometry" governing the structure of the respective space. This geometry does not itself consist of "meaningful" propositions in the sense defined *within* the respective space. Its propositions are what Wittgenstein will later call grammatical propositions.

Chapter 4

The "Middle Wittgenstein": Disintegration of Space

> [...] *daß alles Denken im Räumlichen vor sich geht, daß der Denkprozeß eine Verquickung unsagbar verwickelter vieldimensionaler logischer Räume darstellt, diese Theorie besitzt allergrößte Wahrscheinlichkeit.*
>
> Hermann Broch
> *Die Schlafwandler*

We have seen that one essential feature of the *Tractatus* account of logic and therefore also of the central metaphor of "logical space" was its unity or "logical absolutism" (Friedman 1999, 183): that there is *one* logical structure underlying all kinds of languages, a general form of the proposition. It is clear that Wittgenstein regarded this claim for universality as one of the "grave mistakes in what I set out in my first book", as he puts it in the preface of the *Philological Investigations*, his second book, where he addresses this point explicitly: "We see that what we call 'proposition', 'language', has not the formal unity that I imagined, but is a family of structures more or less akin to one another" (PI, §108). He is referring here, of course, to his notion of "family resemblances" that has been introduced earlier in the book and which is one of the most powerful passages against the unifying tendencies Wittgenstein aims to overcome:

> Consider, for example, the activities that we call "games". I mean board-games, card-games, ball-games, athletic games, and so on. What is common to them all? – Don't say: "They must have something in common, or they would not

> be called 'games"' – but look and see whether there is anything common to all. – For if you look at them, you won't see something that is common to all, but similarities, affinities, and a whole series of them at that. To repeat: don't think, but look! [...] And the upshot of these considerations is: we see a complicated network of similarities overlapping and criss-crossing: similarities in the large and in the small.
>
> I can think of no better expression to characterize these similarities than "family resemblances"; for the various resemblances between members of a family – build, features, colour of eyes, gait, temperament, and so on and so forth – overlap and criss-cross in the same way. – And I shall say: 'games' form a family. (PI, §§66–67)

In this chapter and the following, I will substantiate my claims about continuity and discontinuity in Wittgenstein's philosophy which are related to this passage: I will argue in sections 5.3.2 and 6.1.2 that it is no accident that he chose "games" as the example for family resemblance concepts, but that game-like frameworks replace space-like structures as the most important analogy to language use; in the present chapter I will focus on the role of "spaces" in the so-called middle-period which roughly stretches from the completion of the *Tractatus* in 1918 to the first drafts of the *Investigations* in 1936–7. While I will highlight the differences between games and spaces in chapter 6, I shall first focus on their similarities, notably their non-universal nature and the features they inherit from the Tractarian "logical space" carved out in the last chapter.

An early trace of the idea of family resemblances can be found in a picture from the 1920s made by the Viennese photographer Moritz Nähr under Wittgenstein's guidance. It is a superposition of four portraits in the manner of Francis Galton's composite photography. The portrayed are Ludwig himself and his three sisters so that the picture shows indeed a family resemblance in a literal sense without being a sharp picture of one particular characteristic they all have in common. (cf. Nedo 2012, 268). To be sure, this picture is no evidence of an elaborated philosophical concept, but it clearly belongs to the cluster of problems that occupied Wittgenstein in the 1920s and in his first notebooks after the *Tractatus*: Wittgenstein explicitly mentions Galton in philosophical contexts in 1929 and 1930 (*Lecture on Ethics*/ TS207, 2; MS107, 175); I will also show that in this period, he uses "space" as a family resemblance concept which serves as a means to clarify different domains of language through a web of similarities and differences, but without having one feature that goes through all of them.

In this chapter, I explore how the move from logical absolutism towards a more pluralist account of language takes place in the middle-period and how this is relates to the notion

of space and geometry. I start by sketching the development of Wittgenstein's idiosyncratic notion of "grammar" which evolves in that time; comparisons to geometry pervade this early concept of grammar and I will trace these analogies and evaluate them in light of the role of space and geometry in the *Tractatus*. Following these general observations about the importance of spatial imagery, I move on to specific applications of this geometric-grammatical thinking in various philosophical fields. One obviously related subject is mathematics where geometry has its proper place; another one is the grammar of colour expressions which is of great importance in Wittgenstein's philosophy at all stages (cf. Rothhaupt 1996) and is often regarded as a milestone in his development from the *Tractatus* to his later philosophy.[86] In 1929 he is also said to have had a "phenomenological phase" (cf. Kienzler 1997; Park 1998) in which a domain of immediate experience is contrasted with a more mediated and hypothetical "physical language". Finally I will consider the issue of intentionality, that is, the connection between mind and world which Wittgenstein also interprets in a "grammatical" way: language sets up the structure in which thinking and reality meet; apart from that no metaphysical connection between them is possible, especially no immediate perception of "facts" as it was envisaged in the phenomenological phase.

4.1 Grammar as Geometry

In May 1930 Russell was asked to evaluate Wittgenstein's latest work in order to support his application for a fellowship at the Council of Trinity College. Besides explaining his ideas in conversations, Wittgenstein also created a typescript, a synopsis of what he had written since 1929 which later became the basis of the posthumously published *Philosophical Remarks*. Russell commented on this typescript in a letter to Moore and, more formally, in a report to the Council of Trinity College. His judgement is "a bit of a caricature" as Schulte (2006, 566) observes, but it does capture an important aspect:

> [Wittgenstein] uses the words 'space' and 'grammar' in peculiar senses, which are more or less connected with each other. He holds that if it is significant to say 'This is red', it cannot be significant to say 'This is loud'. There is one 'space' of colours and another 'space' of sounds. These 'spaces' are apparently given a priori in a Kantian sense, or at least not perhaps exactly that, but something not so very different. Mistakes of grammar result from confusing 'spaces' [...] One might define a space, as he uses the word, as a complete set of possibilities of a

86. Peter Hacker famously stated in the first edition of his *Insight and Illusion*: "Wittgenstein's early philosophy collapsed over the inability to solve one problem – colour exclusion" (1972, 86).

> given type. If you can say 'This is blue', there are a number of other things you can say significantly, namely all the other colours. (Russell 1968, 198)
>
> There are various relations among colours which constitute the geometry of that 'space'. All this is, in one sense, independent of experience: that is to say we need the kind of experience through which we know what 'green' is, but not the kind of through which we know that a certain patch of wall is green. Wittgenstein uses the word 'grammar' to cover what corresponds in language to the existence of these various 'spaces'. (200)

In fact, the idea to account for possibilities with spatial structures was not new. Logic was compared to geometry in the *Tractatus*: a logical place, like a geometric place, was the possibility of an existence; logical propositions, like geometrical propositions, did not say anything about the world; logical space, like geometrical space, was a framework of possibilities. Crucially, there was a difference between space as a structure and things or propositions *within* space. Remember the Kantian philosophy of space which motivated Russell's comparison of grammar to a kind of "a priori": geometry studies space itself as a pure form of intuition. But what apparently struck Russell as central in the writings from 1929 and 1930 is the connection between spaces and grammar.

It is sometimes said that grammar is the "heir of logical syntax" in the *Tractatus* where "heir" is meant to highlight the modification and extension of the concept (Hacker 2013b, 256). This reading seems plausible as "syntax" and "grammar" stand in a close, hyponymous or even synonymous,[87] relation in ordinary language. However, what exactly logical syntax is is even less clear than in the case of logical space for which I have provided some reference points in the last chapter. It tends to occur in lower numbered remarks and its function seems to be to explain the logical possibilities of language, but the concept itself is not explained. Let's consider its occurrences:

- the rules of syntax guarantee that signs are used unambiguously (TLP, 3.325)
- its scope is restricted to the combinatorial possibilities of signs, it is not concerned with their *meaning* or *reference* (*Bedeutung*; TLP, 3.33)
- its rules must go without saying "once we know how each individual sign signifies" (TLP, 3.334)

87. In the *Tractatus*, where the earliest mention of grammar can be found, they seem to be synonymous. A symbolism which uses its signs unambiguously is described as one "which obeys the rules of *logical* grammar—of logical syntax" (TLP, 3.325). By emphasising the word "logical" Wittgenstein wants to distinguish the logical grammar from ordinary grammar (and logical syntax from ordinary syntax).

- its rules determine the possibilities to substitute symbols by other symbols (TLP, 3.344)
- if the rules of any sign language are known all propositions of logic are known, too (TLP, 6.125)

Hacker acknowledges the difference to grammar which is a much richer concept and indeed central to Wittgenstein's post-Tractarian philosophy (2013b, 256–257); I shall not dwell too much on this interpretation of syntax as it seems to me very obscure and also leads to confusion with Carnap's theory of the same name. Instead, I would like to suggest following Russell's intuition (which is indeed not very surprising in light of the material of the *Philosophical Remarks*) that there is an important connection between grammar and space, and I aim to substantiate it by taking into account what I have said about geometry and logical space in the previous chapters[88] and also by considering material that was written later than 1930.[89] My account is not strictly chronological as the notion of grammar develops waveringly in this time; but one can see how early ideas about what grammar is supposed to account for, are resumed in the later more elaborate remarks on grammar and its related concepts. Let us recall the functions of logical space from the last chapter: it is meant to provide a framework of possibilities, and thereby of necessity; related with this, it was an illustration of negation and other logical operations; it was meant to be complete (propositions "reach through the whole space") and universal (there is only one logical space, or indeed, there *can* be only one, it is "not even *one*-fold", "auch nicht einmal *ein*fach"; MS157a, 47r). In this section, I focus on possibility and completeness, negation will come up in the subsequent sections 4.3–4.5.

While "grammar" occurs only once in the *Tractatus*, it is very present from the start of Wittgenstein's writings after 1929 where he seems to outline a number of problems that shall occupy him over the years to come. Among them is the question of the status of phenomenology which oscillates in this year between the description of immediate experience in a psychological, pre-logical, sense and a grammatical understanding which

88. This connection has to my knowledge not been studied in detail, although it is sometimes mentioned casually, e.g. by Schulte who comments on a remark by Wittgenstein on William James and his notion of "a priori": "Among other things it shows that "grammar" is conceived as a kind of space, or as analogous to space, and in this respect reveals itself to be a successor to logical space which figures prominently in the *Tractatus*" (2017, 319).

89. *Philosophical Remarks* appears to be a proper work by Wittgenstein as it features a certain completeness, a title and a motto, but biographical evidence speaks against the assumption that it was ever meant to be a book. The posthumous publication is based on TS209 which is composed out of rearranged cuttings from TS208 which in turn was dictated from MSS105–108. In the manuscripts, there is no break between the last remark that was dictated in TS208 and later remarks (MS108, 133). I would like to highlight that the *Philosophical Remarks* are basically a snapshot of Wittgenstein's philosophical development whose scope is contingent: he did not decide to submit the typescript because he saw it as a completed, substantial "work", but because he had to provide a writing sample for a fellowship application.

is not yet specified at this stage. However, this grammatical reading – which "wins" over the psychological reading: in 1933 he writes "Phenomenology is Grammar" (TS213, 437) – is already contrasted with physics in these very first remarks after Wittgenstein's return to philosophical writing:

> Physics is distinguished from phenomenology by its aim to state laws. Phenomenology only states possibilities
>
> Then phenomenology would be the grammar of those facts on which physics builds its theories. (MS105, 5)

The relation between phenomenology and physics shall be considered in more detail in section 4.4, but its basic opposition can already be stated: phenomenology *qua* grammar is concerned with possibilities, not with facts, with *sense*, not with *truth* (cf. MS105, 3). This "grammatical method" Wittgenstein increasingly regards as his own method:[90]

> *My way of philosophising is still and over and over again new for me and this is why I have to repeat myself so often.* [...] This method is basically the transition from the quest for *truth* to the quest for *sense*. (MS105, 46; first sentence in code)

We already know this distinction from the *Tractatus* and we have seen that it belongs to the general question of possibility and necessity which is conceptualised in terms of spatial metaphors. If we remember that the notion of "logical place" was introduced in order to account for the capability of language to express a fact whether it exists or not, and that this analogy grew into a whole structure of "logical space", we can easily see the continuity to the new concept of grammar. Grammar accounts for possibilities, not facts, and therefore features the same necessity as logical space: it is a necessity that "rides on the back of contingency" (cf. Gerrard 2017, 159) as it constitutes the totality of contingent possibilities. Emphasising the difference of this *a priori* method to an empirical investigation of facts, Wittgenstein objects to Mach's usage of the term "thought experiment" who generalises the word "experiment" from science to other spheres in which possibilities are explored and clarified, for example by an artist or a businessman (cf. Mach 1976, 136).

> What Mach calls a thought experiment is of course no experiment at all. It basically is a grammatical investigation. (MS107, 284 f.)

90. The title of Rush Rhees's publication *Philosophical Grammar* goes back to a remark in the manuscripts where Wittgenstein considers it as a title for his envisaged book. It is also the title of two manuscript volumes (MSS113; 114).

It is not an "experiment" because it is not empirical and *a posteriori*. (Likewise, the relativisation of geometry was not the result of an experiment. The necessity of Euclidean space was not refuted empirically, but by mathematical considerations; only on the basis of Riemann's work could the question for physical geometry become empirical). In a way, one already knows what one wants to find out in a thought experiment, it is only made explicit in thinking. This is related to a view that Wittgenstein holds at all stages of his philosophical development: there can be no surprise in logic (in 1918: TLP 6.1251; in 1930: MS108, 137; in 1938: TS221a, 204–5[91]).

The characterisation of Wittgenstein's own grammatical method in terms of the figuration of empirical *possibilities* remains valid in the *Philosophical Investigations* where it is stated in a central passage at the beginning of the so-called "philosophy chapter":

> We feel as if we had to *see right into* phenomena: yet our investigation is directed not towards *phenomena*, but rather, as one might say, towards the '*possibilities*' of phenomena. What that means is that we call to mind the *kinds of statement* that we make about phenomena. [...] Our inquiry is therefore a grammatical one. (§90)

Besides this focus on possibility, it is the completeness of space that is taken up by the middle Wittgenstein's grammar. What appeared to be independent elementary propositions is now described in terms of grammar as we will see in the section on colours. What stays the same, however, is the idea of a logical or grammatical space which has to be *complete*. This again, is most obvious in the colour example in which one colour excludes all other colours for the same object at the same time. But the relations between propositions within a grammatical space need not be rules of exclusion. Any sort of relation between expressions that bears the kind of necessity that is typical for spatial propositions, i.e. that one of its possibilities *has to* obtain, can be described by a grammatical proposition. In turn, the grammatical propositions, which are true no matter what, function like a system of dimensions which determine the possibilities of empirical actualities; they constitute the meaning of a word for the middle Wittgenstein. The meaning of red is constituted by its place in the space of colours, which includes its incompatibility with other colours at the same time and place as well as its relation to other colours: pink is lighter than purple is a grammatical proposition, not an empirical truth. (I shall say more on this in section 4.3).

This systematic character of domains of grammar (together with the above-mentioned possibility aspect), which is related to the fragmentation of the *one* logical space into many

91. Here with an explicit reference to the difference between real experiments and logical inference: "if that surprises you then you haven't understood it. For surprise is not legitimate here as it would be for the result of an experiment. *There* – I would like to say – you may succumb to the appeal of surprise – but not if your experience is at the end of an inference".

grammatical spaces with their own grammatical rules, is what motivates Wittgenstein's frequent comparisons with space and geometry.

> The meaning of a sign is the whole symbol to which the sign belongs.[92] Or one could say it is the place in grammatical space at which it stands.
>
> In some sense one can say: A propositional sign has sense only in the system of a language. (MS109,174 f.).

> Grammar describes the system, the space, one place of which the symbol/sign points to. (MS110,126)

> Meaning, the place of a word in grammatical space (TS213, 34)

To stress that logic is *a priori* and independent of particular natural or human conditions he states that "logic is a geometry of thinking" (MS108, 242) – which also shows that logic and grammar belong together. Mathematics, *qua a priori* method an investigation of possibilities of transformations, could hence also be conceived in terms of grammar:

> Every mathematical investigation is quasi an investigation of space. That one can investigate things in space is clear. But to investigate space itself!
>
> (Geometry and grammar always correspond to one another) (MS108, 27)

This extended concept of space as a system of possibilities is the background for the various applications Wittgenstein makes of space, geometry and grammar in the writings after 1929.

4.2 Mathematical Spaces

The very first remarks after Wittgenstein's ten year absence from philosophy put the assumed unity of number space into question (02/02/1929):

> Is a space conceivable that contains only all rational points, but not the irrational ones.
>
> And this only means: Are the irrational numbers not already predetermined by the rational ones? (MS105, 1)

92. Generally, Wittgenstein's terminology in these writings is not always consistent due to his developing philosophical views at that time. Here, however, he seems to deliberately pick up the sign-symbol distinction from the *Tractatus*: the sign alone does not have sense, but through its possible applications it has sense and thereby is a symbol (cf. TLP, 3.326). The space of possible applications is what Wittgenstein here calls "grammatical space".

Over the following years this question shall come up again and again, crystallising more and more clearly in the insight that different rules apply for different kinds of numbers. The order to write down *all* digits of a number makes sense for normal decimals, but it does not for irrational numbers. Likewise, a question like "does the sequence 7777 occur in π?" (which can perhaps be answered "yes" if it does occur in the development as far as we know it – but what if it doesn't? Can we then answer "no"?). How can we define irrational numbers with exactitude? Are methods like Dedekind's cuts satisfactory or is it not equally inconceivable to infinitely approximate a number that always eludes a final grasp? Can such a number be called defined in the same sense as, for example, integers or even periodic decimals are defined? When resuming these remarks in 1931, Wittgenstein's answer to the initial question is clearly "no" (and this answer bears an interesting hint towards the further fate of the space concept):

> Is a space conceivable that contains only all rational points, but not the irrational ones?
>
> And this only means: Are the irrational numbers not already predetermined by the rational ones?
>
> No more than the game of chess is predetermined by the game of draughts.
>
> The irrational numbers do not fill a gap that is left open by the rational ones. (MS111, 29)

Comparisons between mathematics and spaces or games permeate Wittgenstein's thinking until his death and he keeps on using these analogies when considering philosophical problems in other fields. Scholars like Hilmy and Kuusela argue that, contrary to widespread opinion, Wittgenstein's conceptions of language as a calculus, which came up in the 1930s, are not merely confused attempts to get at the essence of language, but that calculi and language-games serve a similar purpose.[93] I agree with this and would add that both also fulfil functions that resemble those of Wittgenstein's notion of space, that is, first, representing possible connections or actions in a given domain and, second, distinguishing one space from another in which other rules apply. The calculus for rational numbers is different from calculi that work for irrational numbers. Likewise, a general expression like "all points of a straight line" belongs to a different space than a disjunction of concrete elements of a finite set.

> It is one calculus to which our account of generality belongs and another in which there is that disjunction. When we say a cross is located between two

93. See especially note 123 in Kuusela 2008 where he mentions proponents of a widespread reading (Hacker, Schulte, Kenny) and argues against them that, in the middle period, calculi are only used as objects of comparison. Hilmy provides a similar argument in ch. 4 of Hilmy 1987. See also section 5.3 of this thesis.

> lines, then we do not have a disjunction ready which could take the place of this general proposition (TS213, 313).

Other calculi might regard the differences between algebra and arithmetic.

> Could one say: in arithmetic we do not need the associative law, but we work only with specific number calculations
>
> And algebra, even if it makes use of arithmetic notation, is a totally different calculus and cannot be deduced from the arithmetic one. (TS213, 723)

The point is that Wittgenstein uses the word calculus in a sense very similar to space which explains his explicit use of space metaphors when talking about mathematical frameworks. It is the same function as in other spheres, namely to account for a possibility of moves.[94] Since in mathematics this is obviously not a preparation of *empirical* moves, it is a particularly good example to show the apriority of such geometrical frameworks.

Mathematical propositions were Kant's prime example for synthetic truths *a priori* and Euclidean space was taken to be a necessary and universal form of intuition, thereby providing a bridge between mathematics and physics. The development of non-Euclidean geometries in the 19th century and their successful application in general relativity, destroyed this bridge and led to new questions concerning the possibility of *a priori* knowledge: if a direct intuition of space and time is not possible, is the status of constitutive principles merely conventional or can they, in the end, be reduced to empirical laws? Wittgenstein did acknowledge the temptation to conceive of mathematical insights in terms of a "synthetic a priori"; however, he would not regard such an insight as *true*, but as the invention of a new mathematical space, a new calculus. That the mathematician is "not a discoverer, but an inventor" (TS221, 215) is a famous and often-quoted Wittgensteinian slogan against mathematical Platonism; it is illustrated by the conception of various mathematical systems, calculi, or spaces. The answer to a mathematical question is for Wittgenstein not the solution of a puzzle within a given framework, but the imagination of new frameworks and systems. In a remark that contrasts mathematical, grammatical, investigations with geographical expeditions (this resembles the contrast between thought experiments and experiments), it is stressed that a mathematical investigation is not like a search within a given space.

> How strange it would be if a geographical expedition did not know whether it had a goal, and thus also whether it has a way at all. We cannot think this,

94. That a calculus constitutes moves instead of locations is a significant difference to spaces, but here I focus on their similarities. In chapters 5 and 6, I shall investigate the intermediate character of calculi between spaces and games.

> it yields nonsense. But in the mathematical expedition it is precisely like that. Hence it would perhaps be best not to make this comparison at all.
>
> It would be like an expedition that was not sure *of space itself*! (MS108, 92)

Much of this centres on the question whether mathematical propositions are analytic or synthetic. A mathematical problem, Wittgenstein says, is not searching within a system, but the invention of a new system or the explication of a system that exists in a different form.

> The difficult mathematical problems are those for whose solution we don't yet possess a *written* system. The mathematician who is looking for a solution then has a system [...] "in his head" and endeavours to get it down on paper. Once that's done the rest is easy. But if he has *no kind* of system, either in written or unwritten symbols, then he can't *search* for a solution either, but at best can only grope around. – Now, of course, you may find something by random groping. But in that case you haven't searched for it, and from a logical point of view, the process was synthetic; whereas searching is a process of analysis. (MS105, 24)
>
> One can only search for something within a space (MS107, 151)
>
> One can only search for something within a system; hence there necessarily is something that cannot be searched for (MS108, 11).
>
> Mathematics essentially works synthetically (MS106, 285).

The development of new symbolisms can be considered the invention of a "new space" in which different rules apply, for example Sheffer's "discovery" that disjunction and negation can be expressed in one symbol, the Sheffer stroke (cf. MS108, 106), or the insight that certain fractions like $\frac{1}{3}$ are periodic as decimals. This last insight is explicitly called a "new calculus" and compared to what Kant meant when he called "$5+7=12$ not analytic, but synthetic a priori" (MS114, 10v–11r). Later, Wittgenstein would add on this issue: "what makes one speak of a *synthetic* proposition is the *new* form" (MS125, 79).

He seems to make a distinction that is comparable to Kuhn's distinction between "puzzle-solving" normal science and "revolutionary", paradigm-changing science (Wittgenstein stresses the conceptual, rather than empirical character of the latter). Confusingly, here he uses "discovery" in the sense of invention which he later wants to explicitly distinguish from "discoveries"; the purpose of space as a metaphor is clear enough, though:

> The discovery of the connection of two systems was not in one space with those two systems and if it had been in the same space, then it would not have been a discovery (but the solution of a school exercise).

> Where there is now a connection that was unknown before, there was no gap, or incompleteness before which has now been filled. – (At that time one could not say "I know the issue up to this point, from here on I don't know it anymore") (MS108, 19–20)

And later, in 1937, these systems are explicitly referred back to geometry with its features of constituting possibilities which is also why these propositions appear to be necessary (this will become important in *On Certainty* where mathematics also serves as an analogy to language-games).

> When we have a *system* of mathematical propositions, it has its own geometry (MS121,76)

> The logical certainty of proofs – I want to say – does not go further than its geometrical certainty. (MS122, 84)

4.3 The Grammar of Colour Space

The problem of colour incompatibility is already discussed in the *Tractatus* where it is mentioned as an example of logical impossibility.

> For two colours, e.g., to be at one place in the visual field, is impossible, logically impossible, for it is excluded by the logical structure of colour. (TLP, 6.3751)

Wittgenstein compares this to the rule that a physical particle cannot be at two places at the same time which is explained in the underlying notebook with the "structure of space and of the particles" (NB, 18/08/1916). It may be asked what this "structure" is supposed to be, but I am quite sure that it must mean what Wittgenstein elsewhere calls "form".[95] As seen in chapter 3, Wittgenstein conceives of the "form" of colour and space as "spaces of possible atomic facts" which thereby delimit, *a priori*, the range of combinatorial possibilities for objects.[96] When the problem returns in 1929, he indeed uses "form" instead of "structure". But it is then dawning on him that there is no logical contradiction in the usual sense in the statement under question, an objection which has been pointed out to him by Frank Ramsey.

95. In most discussions, philosophical or not, these two are used interchangeably. In the terminology of the *Tractatus*, it would, however, be more consistent to use "form" in this case because its relation to structure is one of possibility–actuality and what he needs in his argument is *a priori* possibility.

96. The comparison between coloured objects and objects in space also shows that both "forms", colour and space, are similar in a way: they are both manifolds of possibilities. Since the claim of the *Tractatus* was that each of these manifolds is also logical and hence there is a logical space which accounts for all possible atomic facts, the specific rules of these manifolds such as colour-exclusion had to be "logical impossibilities".

> That two colours cannot be at the same time at the same place has to be determined by their form and by the form of space. (MS106, 81)

> No one could say that inference from 'This is red' to 'This is not blue' was formally guaranteed like the syllogism. (Ramsey 1990, 48)

In light of Ramsey's criticism, Wittgenstein comes to admit that some necessities cannot be explained with the assumed universal logic of the *Tractatus* in which all elementary propositions were supposed to be independent. The model case, which occurs most in the manuscripts and is discussed a lot in the literature,[97] is the necessity (or impossibility) that one thing cannot be red and green all over at the same time. Given that two propositions exclude one another, they have to be further analysed according to the *Tractatus*: the logical product (conjunction) of "this object is red" and "this object is green" yields a contradiction and therefore these two propositions cannot be elementary (cf. TLP, 6.3752). The analysis must show that "this is red" *entails* "this is not green" so that the conjunction with "this is green" is a contradiction of the form p.~p. Wittgenstein tried to provide such an analysis in 1929 – and failed. He came to the conclusion that there must be a kind of impossibility other than contradiction and this kind of impossibility must result from the rules that govern the specific domain in which we are operating (cf. SRLF[98]). The domain of colours, like basically all domains that include measurement in any form, demand something like a "Principle of Single Value" (Chang 2008, 123): a rule which states that each coordinate can only be assigned once, an object can only have *one* length, *one* weight, *one* colour etc. More domain-specifically, the relations between the colours, e.g. that orange is closer to red than to green, should also be entailed in these context-sensitive rules, because they are equally *a priori*. Wittgenstein soon comes to speak of a "colour space" that serves this purpose and introduces other phenomenological spaces such as "pitch space" and "sensational space".[99]

These domain-specific logical spaces, or as he now calls them: grammatical spaces, have an impact on Wittgenstein's notion of negation. He still holds that a proposition and its negation cover complementary regions in a complete grammatical space (after all, this amounts to the "Principle of Single Value": a proposition "this is red" excludes the whole rest of colour space); but this only accounts for the very specific space we are working in, for a

97. To name just a few authors who cover this issue: Hacker 1972; Hintikka and Hintikka 1986; Rothhaupt 1996; Kienzler 1997; Monk 2014; Engelmann 2013b; Ometiţă 2017; Westphal 2017.

98. It is significant that *Some Remarks on Logical Form*, the only article Wittgenstein ever published (1929), deals with "logical form" in the Tractarian sense, that is with *a priori* possibilities for which he now seems to acknowledge some kind of empirical constraints (see also section 5.2.1).

99. This intuition is already present, but not elaborated in the *Tractatus* (cf. TLP, 2.0131). Later he admitted that this remark could already have led him to the insight that a "coordinate" in such a space can only be assigned once (cf. MS108, 53).

particular domain of language. To say that something is not red, means that it can be orange, green, blue etc., but it would be strange to say: "it is not red, it is heavy".[100] The proposition still "reaches through the whole logical space" (TLP, 3.42), but this whole space is limited to the domain of, in this case, colours and does not apply to length or any other domain (cf. Ometiţă 2018, 148–149). Hence, negative statements are not completely independent of their positive counterpart, but constitute a kind of "zero-point" of a dimension that is shared with the positive proposition, as Wittgenstein shows in one of his preferred fields of interest, "pain-space":

> 'I have no stomach ache' is comparable to 'These apples cost *nothing*'. For they cost no *money*, but not no snow or no effort. The zero-point is a point on *one* scale. And as no point of a scale can exist without the scale, nor can the zero-point. [...] I compare this state [*i.e. not being in pain]* to another state, hence it must be comparable. It must lie in pain-space as well, but in another place. – Otherwise my sentence would say that my current state had *nothing to do* with a painful one; as in: the colour of this rose has nothing to do with the conquest of Gaul by Julius Caesar. (MS108, 38)

The idea of contextual spaces is still present in the *Philosophical Investigations*, albeit not explicit. They underlie, for example, the remarks on ostensive definitions in (roughly) §§28–37 which can perhaps be summed up as follows: one already has to know in which space one is operating in order for an ostensive definition to have sense. Similarly, orders like "Look at the *colour* of the vase" or "look at the *shape* of the vase" might induce superficially similar actions, but they surely have different meaning and purpose for they set up different grammatical spaces (PI §§33–34). Consider also Wittgenstein's comment on intentionality much later in the *Investigations*: "if I say falsely that something is *red*, then, all the same, it is *red* that it isn't"[101] (PI §429) – here I am operating in colour space or, for that purpose, in the "space of redness" which only distinguishes between red and not red – a binary space similar to the drawing from November 1914 (cf. chapter 3). Saying that something is not red is not completely detached from any redness, but has something to do with it. Determining a grammatical space, as it is primarily exemplified by means of colour space, is determining a domain of language and the specific rules that are valid in it. It is akin to establishing a

100. A sensible use of this sentence is conceivable: but in that case we would rather be talking *about* spaces, not about objects *in* spaces. It would basically be pointing out that someone is committing a category mistake.

101. In German it goes "daß, wenn ich fälschlich sage etwas sei *rot*, es doch immerhin nicht *rot* ist". This construction is a bit complicated in English, but the important point is that the emphasised "*red*" determines the topic of the sentence, it determines the space or the measure that is used. Even if the sentence is false and nothing is red in reality we are still talking about this colour; a positive statement about its absence would point to the "zero-point" of this measure.

measure with which we can then compare reality and even if the measured object has zero units it has still been measured on that scale.

4.4 Phenomenological vs. Physical Spaces

Immediately after the remarks on mathematical spaces in February 1929, Wittgenstein reflects on the nature of visual space and how it could best be represented. He considers physics and phenomenology as possible candidates and makes a crucial distinction: physics strives for truth, facts and laws, phenomenology for sense, possibilities and grammar. "Phenomenology would be the grammar of the description of those facts on which physics builds its theories" (MS105, 5). The distinction between grammatical statements about possibilities and empirical statements about facts is a constant in Wittgenstein's philosophy early and late. It takes up the possibility-actuality distinction that was latent in Kant and central to the *Tractatus*. The specific result of Wittgenstein's so-called "phenomenological phase" is that phenomenology is classified as grammar[102] and stripped of any psychological vestiges that seemed to be a necessary requirement for the description of context-specific rules in 1929 (cf. Rothhaupt 1996, 69–81; Kienzler 1997, 105). In this year, he had tried to find a "phenomenological language" that would immediately represent sense experience and thereby reveal hidden rules like that of colour exclusion. However, he came to acknowledge that such a language was neither necessary nor possible.

Two problems of this envisaged phenomenological language led to its rejection: exactitude and time. Both are related insofar as they both stem from the opposition of an unfathomably rich experience and a language that seems to be too coarse, too inexact, too static to describe it. Describing the fleeting reality seems to always falls short of the actual experience. If I look up to a starry sky for a moment and then try to describe how many points of light I have seen, I do not know what to say. On the one hand there supposedly must be a definite number of points that I have seen, on the other hand there is no way how I could access or verify this number. The problem is the result of a misunderstanding, says Wittgenstein, namely the comparison of the memory image with an actual, physical image to which I could go back and count. Saying that language captures reality "only" roughly seems to suggest that an absolute exactitude is, in principle, possible – but how should this ever be reached? Wittgenstein's peculiar kind of verificationism, which comes up in this time, consists in this: that the way a proposition can be verified defines a grammatical space in

102. One of the chapters of the *Big Typescript* (1933) is even entitled "Phenomenology is Grammar".

which alone the proposition has sense. The grammar of a visual experience is different from the grammar of a painting as physical object.[103]

Parallel to the mathematical discourse on different spaces for discrete rational numbers and elusive irrational numbers, Wittgenstein contrasts stable physical objects and elusive objects in the visual field. His attempts to provide a foundation of knowledge by finding a language that is free from any hypothetical assumptions fail because such a language would operate in a categorically different sphere than our hypothetical physical language. Statements of equality and exactitude have a different meaning in visual space than in Euclidean space. And although Wittgenstein does speak of visual space as geometry, he does not think of this non-Euclidean geometry in the sense of a curved space,[104] but as geometry in the sense of grammar, a set of rules which constitutes the meaning of expressions.

> When we speak of visual space we are easily deluded by the idea that it is a sort of raree-show box which each of us carries with oneself. That is, we use the word "space" [*Raum*] similar to "room" [*Raum* or *Zimmer* in German]. But in fact the word "visual space" only refers to a geometry, I mean to a part of the grammar of our language (MS113,124).

The difference between physical space and visual space is that the geometry of the latter cannot be specified by means of experiments. In physics, measurement can yield surprising results that change our conception of space, whereas the features of visual space are "already in plain view", more akin to the grammar of time of which Wittgenstein says in the *Investigations* that it "has to be *called to mind*" and that it is something which, "for some reason, is difficult to call to mind" (PI, §89). It is probably not by accident that in this central remark of the carefully composed *Philosophical Investigations*, time plays a crucial role: it is a good example for Wittgenstein's method of distinguishing domains of sense because it illustrates the difference between phenomenological and physical spaces. As Augustine, who is quoted in §89, says, we generally know what time is, only when asked about its essence we do not. Confusions arise when one domain of language, one space or one system, is transferred to another without being aware of the change of domain.

103. In the context of counting things, one could say it has a different *arithmetic*: the arithmetic of visual space is more like "1, 2, 3, 4, 5, many" (VoW 2003, 318). Wittgenstein's wider account of geometry allows him to understand arithmetic as a kind of geometry, just like time can be understood as a kind of space (cf. PPF, §70). In the middle period, Wittgenstein calls arithmetic a "kind of geometry [...] one could say: it is a more general geometry". To demonstrate that these different expressions all serve a similar purpose, is part of my project to show the continuity from spaces to games. After the classification of arithmetic as geometry, Wittgenstein writes: "And can I not say that, in this sense, chess is a kind of geometry as well (And indeed any other game too)" (MS108, 116).

104. Helmholtz and later Schlick had already shown that Euclid's axioms are not valid in intuition spaces, such as visual space and sensual space. Wittgenstein would later say that it is non-Euclidean, but not as a result of experiments, but by contemplating its grammar or "internal geometry".

In the 1930s, Wittgenstein contrasts physical and phenomenological time in a simile: the time measured in physics is compared to a film strip running through a projector; on this strip all pictures are ontologically on the same level and what we call "present" is just the picture that happens to be in front of the lamp at a certain point. Time experience, by contrast, is compared to the screen on which the picture is fluid, ever changing and non-segmentable; strictly speaking, it does not allow for different time states past, present, and future as there is only one lived experience consisting of memories, an extensionless, ever moving, present and an unknown future.[105] If one then asks, as Augustine does, how we can measure time at all since the past is not anymore and the future is not yet and the present is extensionless and fleeting, then one is applying the physical picture onto the phenomenological one. Reminding oneself, as Augustine does, that we can and do measure time and how we do it, is a grammatical observation, Wittgenstein says, which can help us to understand what was wrong with the initial question (PI, §90).

By the middle of 1930, Wittgenstein had arrived at a new account of analysis which replaced the Tractarian analysis of propositions into elementary propositions and truth-functions with his new notion of grammatical analysis, meaning the clarification of the specific domain in which we are operating and distinguishing which aspects in this domain are important for the proposition in question and which ones are not. He put this insight at the very beginning of his typescript *Philosophical Remarks* (TS209) which is a significantly restructured summary of his manuscripts from 1929 and 1930.[106] It is followed by the other insight of his phenomenological phase, namely that a language that describes immediate experience as a sort of foundation for the whole logical apparatus is neither necessary nor possible:

> A proposition is completely logically analysed if its grammar is made completely clear: no matter what idiom it may be written or expressed in.
>
> I do not have phenomenological or primary language as I used to call it, in my mind as my goal. I no longer hold it to be necessary. All that is possible and

105. Cf. MS108, 33; MS111, 9; MS112, 128. Note the similarity to Bergson's cinematic metaphors with which he criticised Einstein's reification of time. Wittgenstein's method of grammatical analysis can contribute to their famous debate by granting validity to the arguments of either side, but restricting this validity to the respective domain.

106. Even though the *Philosophical Remarks* should not be treated as a completed work designed for publication, it still has some authority as it was the first text after the *Tractatus* (with the exception of SRLF) which Wittgenstein prepared to be read by others; and it is a more elaborated version of the merely chronological synopsis of his manuscripts that he had prepared earlier (TS208). The main difference between TS208 and TS209 is that remarks are presented in a different order in the latter; therefore it has some significance that he put a remark at its beginning which expresses a notion of "analysis" that is very different to the *Tractatus*.

> necessary is to separate what is essential from what is inessential in *our* language.
> [...]
>
> Each time I say that, instead of such and such a representation, you could also use this other one, we take a further step towards the goal of grasping the essence of what is represented. (PR, §1a– b)[107]

The idea that analysis consists in clarifying the grammar of a particular space survived the extensive revisions that led to the typescripts of the *Philosophical Investigations*. In §46 and the following discussion of simplicity, Wittgenstein argues that compositeness and the idea of indivisible atoms depend on what is stipulated as a standard of simplicity, that is they depend on the specific game that is played.

> Asking "Is this object composite?" outside a particular game is like what a boy once did when he had to say whether the verbs in certain sentences were in the active or passive voice, and who racked his brains over the question whether the verb "to sleep", for example, meant something active or passive. (PI, §47)

The answer is of course: it depends. It depends on how you look at the question, in which space or in which language game the whole discussion is set. The grammatical form of "to sleep" is the active voice, but if we talk about activities in everyday life, no one would count sleeping among them.

And in §72 and §88, the context dependence of exactitude is illustrated with a number of examples that are conceptually the same as the ones in the phenomenological phase: what can count as exact is defined by what will satisfy our demands for exactitude in a particular situation. Some concepts, such as "game" can have an inherent inexactitude which resembles that of visual objects (and irrational numbers in a certain sense) and in these cases it is senseless to ask for greater exactitude (for example to demand an exact definition of the essence of "game" or to complain about the visual field having blurred edges, or about a memory image being diffuse). "Roughly" and "exact" have different meanings whether we talk about being punctual for dinner or about time measurement in the laboratory (PI, §88).

Also the "relativistic" notion of analysis that is expressed in the *Philosophical Remarks* recurs in the *Investigations*. After describing the Augustinean puzzle about time, Wittgenstein calls his inquiry "a grammatical one".

> 90. [...] And this inquiry sheds light on our problem by clearing misunderstandings away. Misunderstandings concerning the use of words, brought about,

107. In 1932 the somewhat essentialist undertone of this remark is corrected: Wittgenstein no longer speaks of the "essence of what is represented", but only of a goal which he wanted to achieve by means of a phenomenological language (cf. MS114, 14).

> among other things, by certain analogies between the forms of expression in different regions of our language. – Some of them can be removed by substituting one form of expression for another; this may be called 'analysing' our forms of expression, for sometimes this procedure resembles taking a thing apart.
>
> 91. But now it may come to look as if there were something like a final analysis of our linguistic expressions, and so a single completely analysed form of every expression. That is, as if our usual forms of expression were, essentially, still unanalysed; as if there were something hidden in them that had to be brought to light. As if, when this is done, the expression is completely clarified and our task accomplished.

To recognise this latter appearance as an illusion is one of the crucial differences between the *Tractatus* and Wittgenstein's writings after 1929. It is an important change in his philosophy and yet in can be understood in terms of a continuity: the conception of language as a space of possibilities. There is not one final analysis of this space, for example, in a phenomenological language which records sense data in logical form;[108] instead there are many spaces which each have their own rules of analysis, including their own *end* of analysis, and these spaces through their multiple connections and similarities build what we call language.

4.5 Intentionality

Intentionality is an issue in Wittgenstein's philosophy in an indirect, but very profound sense. It is indirect and one has to be careful not to equate Wittgenstein's problems of mind-world relation with Brentano's descriptive psychology (cf. Rothhaupt 1996, 76-77). However, it is true that this relation lies at the heart of Wittgenstein's struggles with philosophy of meaning (cf. Hacker 2013a). How does a thought refer to a fact? How does an order relate to its execution, an expectation to its fulfilment, a wish to its satisfaction? What these cases have in common is that the events that are referred to do not (yet) exist and a strict correspondence theory fails to explain what the mind actually refers to when it is thinking, ordering, expecting

108. With this Wittgenstein anticipates parts of Sellar's critique of the "myth of the given" which can be summarised with the claim that non-propositional knowledge, the given, cannot serve as an ultimate foundation of propositional knowledge of facts. Wittgenstein had already pointed out the incommensurability between these spheres in his treatment of phenomenological language. I will investigate in section 5.4, why he would later accept as a "given" something like the "form of life"(cf. PPF §345). With the spatial image in mind, one can say that "forms of life", although their status is ambiguous, are on the level of geometry and not "in space". For "to specify a space is not to specify a place. (to specify a space is in the end to specify a geometry)" (MS110, 40).

or wishing something that does not exist. "But this paradox (which indeed has the form of a truism) can also be expressed in this way: one can think what is not the case"(PI §95).[109]

One solution to this – or at least a good illustration of the problem – lies in the idea of contextual spaces: the remark §429 mentioned above says what it means for something to be not red: it does neither refer to a "something" nor to a "nothing", but to a point in the space of colour, to the place which *can* be occupied by red – it turns out, that the *Tractatus* idea that places are possibilities was already an attempt to tackle the question of intentionality. To insist then, that "red exists because if there were no red, it could not be spoken of at all" misses the point that this sort of existence is different from the existence of a red object; one should rather say "the word 'red' has meaning" (PI, §58) which amounts to declaring it a part of the grammar of colours. The example could be compared to "the point (3,5,1) exists" in a three-dimensional geometry – which says more about the coordinate system than about the point and whether it is assigned a value or not ("a point in space is an argument place", TLP 2.0131). Before finding its way into the *Investigations*, this idea is elaborated in 1930 when Wittgenstein considers the relation of expectation and fulfilment as an example of intentionality:

> Expectation is not an imagination, for if I expect that a light point will appear, then there must be a place *there* at which the point really will appear if it appears.
>
> Anticipatory in the expectation as well as retrospectively in the fulfilment and memory of the expectation, expectation and fulfilment are in the same space. (MS108, 185)

> Hence, the idea would be that what expectation has in common with reality is that it refers to a different point in the *same* space (space understood very generally). (MS108, 264)

One point where intentionality intersects with the mathematical and the phenomenological questions discussed in the same manuscripts is generality: an expression of an expectation of someone entering the room can be fulfilled by a number of events – but it would be nonsense to say that all these events were somehow included in the expectation. Their "harmony" rather lies in the fact that the same expression is used in both the expectation and in the description of the event. These considerations have a basis in Wittgenstein's rejection of some aspects of

109. The problem is in fact an old one: in Plato's *Theaetetus*, which is repeatedly cited in Wittgenstein's manuscripts (and also in PI §518), Socrates leads his interlocutor into this aporia by letting him accept the premises that 1) a person who thinks always thinks something, 2) who thinks something thinks something that exists, 3) to think what is not is to think nothing, 4) to think nothing is not to think at all: "In that case it's not possible to believe what is not" (Theaetetus, 189a–b).

set theory, especially concerning infinite sets: when one refers to "all elements" of an infinite set, this should not be understood as a disjunction of its elements, i.e. as a disjunction of an infinite number of discrete elements. Instead this kind of generality is to be understood as a rule which has nothing to do with the number of individual elements that fall under its general concept. An interval of a line does not consist of an infinite number of points, but rather constitutes a new space which does not deal with the mathematical problems of continuity and infinity or infinite divisibility. "Generality is not in measuring space" (MS111, 37). "The line is between 5 and 10 cm long" is only considered a question of yes or no, not a question of the exact length of the line.

These considerations are important, because they combine the idea of context-dependent exactitude and the idea of a relative space of agreement (as opposed to a supposed identity of expectation and fulfilment in an absolute space). The expectation that the postman brings the mail can be met by any event that can be described as the postman bringing the mail and is independent from a concrete person whom I may have thought of when uttering the expectation or not.[110] It is a general expectation like the description of an interval or a space whose possibilities in this case are not the distinct instances or points, but are reduced to a question of yes or no, in or out. In trying to grasp the instances of a general description or an infinite set, there would always be an instance that I had not foreseen. To prescind from this infinity of possibilities and instead consider only two possible states, yes or no, is not a logical indeterminacy, but rather a different kind of logic, a different kind of grammar, a different kind of geometry. Wittgenstein sketches structurally similar arguments in numerous occasions, for example:

> I want to say, there seems to be a relation between a point and the edge [of a surrounding area] which is independent of their distance. – It is, as it were, as if I used a geometry in which there is no distance, but an inside and an outside. In this sense, indeed, the pictures  and  are equal. (MS109, 113).
>
> [...] the indeterminacy of generality is not a logical indeterminacy. As if we had not only freedom within logical space, but also freedom to extend and change this space.
>
> Hence not only free mobility, but an indeterminacy of geometry. (MS111, 186)

110. Tim Crane criticised Wittgenstein's grammatical account of intentionality with an argument which basically says that 'Mr Smith brings the mail' can meet the expectation of the postman bringing the mail although I did not think of Mr Smith when uttering the expectation, but of Mr Jones who normally brings the mail to my house. Hence, Crane suggests distinguishing between the object of expectation and how the subject thinks of this object. Since these can be different things, Wittgenstein's account of intentionality is declared incomplete and hence insufficient (cf. Crane 2010).

Wittgenstein repeatedly compares his approach to the intentionality problem with a step that has been made by the theory of relativity.[111] The analogy comes up when he talks about a certain preparation that is made in expecting something, a preparation that sets the stage for the expected event to happen (or not). The question "how can I now in my expectation refer to an event that is not yet there?" is related to the question "how can I now that an event is taking place know that this is what I expected?"; and both are variants of "how can I think something that is not the case?".

> How can I know that this is what I expected if not through the fact that it fulfils my expectation now, corresponds to my expectation now.
>
> A move is required which is similar to that of relativity theory. (MS108, 270)

> The preparation is, as it were, itself the language and cannot go beyond itself (in this not being able to go beyond itself lies the similarity of my observations and that of relativity theory) (MS108, 272)

What resembles Brentano's intentionality, "the mind's direction on its object"', is what Wittgenstein calls the relation or the harmony between thought and world. However for him there is no real agreement, the "harmony" lies only in using the same *expression* in the description of the thought and in the description of the event that corresponds to this thought. The correspondence consists in being describable by the same expression:

> The relation, the connection between thought and reality is reflected in language by them having the same expression. Language cannot represent this relation in any other way.
>
> Here we have a kind of theory of relativity of language (and this analogy is not arbitrary) (MS109, 85)

Later in the same manuscript he expresses the same idea in the context of a discussion of Frege's symbol for assertion: what "I think p" and "$\vdash p$" have in common is only the sign "p", not a third entity to which both refer. "The line of thought that is needed here is again the typical move of the theory of relativity – when I say: that's just how language works". It is again the idea that language, a system of signs, cannot go beyond itself: if I ask something and get a reply (i.e. signs) then this is perfectly fine, and I wouldn't say "but, this is *only* a reply" (MS109, 199).

What these analogies to the theory of relativity hint to is the need to specify a space of measurement in order to make agreement possible in the first place. Like Einstein highlighted

111. For a study on Einstein's influence on Wittgenstein, see Kusch 2011.

the need for definitions in order to give any sense to the concept of simultaneity (cf. Einstein 1920, 19), so Wittgenstein stresses the role of the grammatical space which first of all constitutes the meaning of the terms therein. Like Einstein undermined the idea of an absolute time in relation to which events could be called simultaneous, Wittgenstein rejects the idea of an absolute harmony between thought and reality in a supposed metaphysical realm. "The intention sets a standard according to which the fact can now be evaluated" (MS109, 263). The constitutive function of the stipulation of a measure is not unlike the constitutive function of Kant's *a priori* intuitions, only without the claim for apodictic truth.[112] Evidence that Wittgenstein conceived his analogies to relativity in that way can be found in the contexts of the literal references, for example in 1937. Before reminding himself that "Einstein has taught the world that the method of time-measurement belongs to our grammar of time expressions" he reasons about the agreement of experiences:

> But what is the 'same' experience here? How do we *measure* /compare/ two such experiences /on which scale do we compare them to one another/: after all, this is required to say 'this and this *is* the same experience'! (MS119, 116)

Let me give one more example, this time from the early 1940s when Wittgenstein worked on the foundations of mathematics and rule-following. Again he transfers questions from physics such as the stipulation of a measure – with its background assumptions such as rigidity or constancy – to mental phenomena such as the recognition of a colour. When I see a flower and say "it is red" can I be wrong in this judgement?

> No. The certainty with which I call the colour "red" is the rigidity of my measure, is the rigidity that I take for granted. It is not to be doubted in my description. For this characterises what we call describing.
>
> Following the rule is at the *basis* of our language game. It characterises what we call describing.
>
> This is the similarity of my investigation with relativity theory: that it is, so to speak, an investigation of the clocks with which we measure the phenomena. (MS164, 82)

In all these cases Wittgenstein's account of intentionality, of the relation between mind and world, centres on the constitutive function of the space that is set up *a priori* and which first

112. Michael Friedman has elaborated these historical lines from Kant via the formalisation of geometry to special and general relativity (especially 1999 and 2001). His idea of a "relativized a priori" which goes back to Reichenbach's distinction of the apodictic and the constitutive aspect of Kant's *a priori* forms (Reichenbach 1965, 48), captures an important function of Wittgenstein's grammatical spaces: it constitutes the meaning of certain expressions (and actions) without being universally or necessarily true.

of all determines the candidates for empirical truth. "The grammatical rules first determine the meaning (they constitute it) and therefore they are not responsible to any meaning and insofar they are arbitrary" (MS114, 109).

4.6 Summary

Wittgenstein once considered a line from *King Lear* as a motto for the *Philosophical Investigations*: "I will teach you differences" (Drury 2018, 135) – a certain preoccupation with differences is clearly visible in the period from 1929 to 1937, namely differences between grammatical spaces. It is a move away from an equalising "logical absolutism" to a relativised account. However, I also pointed out what motivates the usage of "space" in different fields: there is a certain continuity in all this diversity. A space is always a framework for the possible candidates for truth. Grammatical or geometrical propositions are therefore not true, but constitutive for the sense of the propositions that can be true. In this they resemble the logical propositions of the *Tractatus*, but, while these had a unique status as applying to *the* logical space, now a plurality of spaces is considered for which different propositions can count as "grammatical". One criterion for a grammatical proposition is for example that it is not possible in the sense of contingent in the *Tractatus*: it is not localisable as a region on the map which would exclude other regions; rather these propositions are valid in the whole of the space (however, without necessarily being formal tautologies). Therefore, grammatical propositions do not have a negation in space like descriptive propositions do. The negation of a grammatical proposition would not be false, a possibility among others which happens to be not the case, but nonsensical insofar as it cannot be described as a possibility "on the map". Hence, two functions of Tractarian space, possibility and negation, are still preserved in the notion of grammatical spaces; like logical space they also have to be complete as we have seen in the case of colours where one colour excludes all the other colours at a certain point at a certain time, and in the case of complete number spaces in mathematics. The flexibility of Wittgenstein's usage of space shows itself in that he considers not only finite and discrete, but also infinite and continuous grammatical spaces. In the latter, the claim for completeness is served by conceiving of grammatical propositions as rules which do not "contain" infinitely many applications, but allow for an infinite possibility, and in the end reduce the space of possibilities to two alternatives: in or out. More on this shall be said in section 6.1.3 when rule-following is considered. The purpose of this chapter was to give an idea of the importance of space concepts in the middle period and of its continuity with logical space.

The frequent use of space metaphors in this time does, despite all similarities, not allow for a straightforward identification of something essentially spatial that is common to all applications:

> Our notation, or mode of expression, shows which similarities – and differences – we want to highlight in particular. So sometimes one calls everything a space that has a similar structure as space, and one always wants to stress this analogy. And then again one only wants to shun this analogy because it leads to confusion and one wants to stress the differences between "spaces" [...] (MS108, 135)

This is something spaces have in common with the family-resemblance concept of games which I will consider, among other things, in the next chapter. For the disintegration of logical space into many grammatical spaces is paralleled by a disintegration of the concept of space into a number of related concepts which serve similar functions. These shall be investigated in the following.

Chapter 5

More Disintegration: Forms and Functions

"Also was sind Richtbilder?" [...]
"Ewige Wahrheiten, die weder ewig noch wahr sind, sondern für eine Zeit gelten, damit sie sich nach etwas richten kann."

Robert Musil
Der Mann ohne Eigenschaften

In this chapter I shall investigate concepts that take up functions of "grammatical spaces", especially that of providing a framework of seemingly necessary propositions which constitute possibilities in a specific domain. I shall *not* focus on the differences between them. Scholars have tended to categorise these concepts in taxonomies in which each is in its place and the differences are emphasised (cf. Baker and Hacker 2009, 273–277; Glock 1996, 135–139). However, while Wittgenstein is indeed a philosopher of differences, he rarely points out *these* differences. He does not explicitly distinguish between different *kinds* of *a priori* framework.[113] Instead, his preoccupation with differences can be specified: First, there are differences between things on the same level which belong to the same space, the same system, and which are determined through their relations to other parts of the system. Second, there are differences between spaces, that is, categorical differences as they have been dealt with in the last chapter. And third, there is the difference between those two differences, i.e. the difference between empirical propositions on the same level and

113. In fact, since sense depends on a sense-constituting framework, and since Wittgenstein rejects any meta-frameworks, it is extremely difficult to talk about them in meaningful words. Therefore, I take it that his various concepts for these frameworks are not to be taken as a sophisticated typology of different frameworks, but as partly metaphorical tools to express various shades of a shared functional role.

grammatical propositions which are not locations in a space, but belong to its geometry. Roughly, the first are empirical differences, the second are grammatical differences, the third is the difference between grammar and experience. Drury's memory of Wittgenstein quoting *King Lear* has the following context:

> Hegel seems to me to be always wanting to say that things which look different are really the same. Whereas my interest is in showing that things which look the same are really different. I was thinking of using as a motto for my book a quotation from *King Lear*: "I'll teach you differences". (Drury 2018, 135)

I take it that e.g. mathematical propositions and grammatical propositions, in Hacker's narrow sense,[114] and "hinge" propositions like "This is my hand" do not look so similar that one has to point out the differences between them in order to avoid confusion. Rather, Wittgenstein in this case wants to highlight the *similarity* between them, their similar role in practice, which allows for the analogies he frequently draws. These analogies do not stress the differences between different kinds of necessity, but between necessity and contingency, between grammar and experience; thereby he aligns propositions that look different, but have similar roles in their respective usage.[115] In this way he even assimilates certain empirical propositions to mathematics (cf. OC, §448), thereby emphasising that they function like non-empirical certainties: he stresses that their necessity derives from their being constitutive of a game. In other places, the same comparison is meant to suggest that even mathematical propositions are not metaphysical truths, but depend on their application – just like the so-called hinge propositions have their extraordinary role due to the fact that they *are* never questioned in our everyday practice, not because they *cannot* be questioned.[116] In my view, this mutual illumination is one of the main motivations for Wittgenstein's intertwining of mathematical and philosophical questions in his writings (more on the ambivalence of constitutive frameworks between *a priori* and experience in section 6.2).

In the following, I investigate concepts that are similar to "spaces" in the sense elaborated so far. By showing the proximity between them, I also make a claim for the continuity of

114. By "grammatical proposition" Hacker seems to mean only rules for the use of language such as "black is darker than white" (cf. Hacker 2012b, 9–10).

115. Without putting too much weight on Drury's recollection, one can still note that Hegel's and Wittgenstein's accounts are not mutually exclusive – rather they have different objects: Hegel is interested in things which look different and Wittgenstein in those which look similar. So it is not impossible that Wittgenstein is in fact quite Hegelian – in this crude sense – with regards to things which look different. My claim is that this is the case with the various and at first sight very diverse topics in his writings which he nevertheless treats in a similar way.

116. Especially in the manuscripts that underly the *Remarks on the Foundations of Mathematics* but also in those that underly *On Certainty*, here again with a reference to relativity theory and its verificationism (cf. §301–307; cf. also section 4.5)

this kind of spatial thinking in Wittgenstein's work. While the word "space" itself becomes less frequent after the middle period, other concepts, notably language-game, take up its functions and remain important throughout the late and latest writings. An important feature that binds together the various concepts I am considering here is their systematicity which is often neglected because the later Wittgenstein seems to be a rather unsystematic philosopher, especially in contrast to his early work. While in his middle period he indeed rejects a universal system, which would integrate the whole of language, he still adheres to *ordered* representations, a kind of local systematicity which I shall present in the first section.

In the second section, I shall look at a variety of concepts for constitutive frameworks that complement the notion of grammatical spaces. This is not a completely new development in the middle period: already in the *Tractatus* some functions of logical space were accounted for by other concepts. The most important one is *logical form* which *a priori* determines the configurational possibilities of objects. Beyond that, "syntax", which later is assimilated to geometry (cf. MS107, 213), and "sense" belong to the Tractarian language of possibility. In the middle period, this plurality of concepts grows further. With the acceptance of a variety of spaces, forms of representation come into focus which are not sub-forms of the *one* logical form as assumed in the *Tractatus*. Similarly, Wittgenstein ceases to speak of the *one* notation which infallibly captures reality (its truth being "unassailable and definitive"), but considers the possibility to introduce notations for particular purposes. A rather surprising candidate for these spatial functions are *hypotheses*, which may become less surprising if we think about Poincaré's conventionalism about geometry and hypotheses which can be elevated to quasi-axioms. Contrary to most readings, I try to provide a place for a positive account of *theory* in Wittgenstein's philosophy which is derived from Hertz's notion of "image"; as I have shown in chapter 3, it is in fact closer to Wittgenstein's space concept than to his picture theory.

While these concepts are important and certainly belong to the orbit of grammatical space, they have a rather supplementary function. To conceive of a constitutive framework as a form of representation, a notation or a theory, may be more appropriate in certain contexts and prevent misunderstandings which "space" brings with it. However, there are other concepts which seem to *replace* space concepts; for this claim I shall give empirical – their occurrences increase while those of "space" decrease – as well as conceptual reasons. These frameworks are especially *calculi*, clearly an intermediate concept, and *games* or more specifically *language-games* which are central in Wittgenstein's late philosophy.

Language-games are, however, embedded in larger frameworks which influence our way of looking at the world as a whole. To these belong the notorious forms of life. It is debatable whether the term is as important in Wittgenstein‘s philosophy as is sometimes suggested,

but they surely have some grounding function for what societies and individuals consider meaningful and „possible“ (possible locations if one wants to use this now a bit obsolete metaphor). Other frameworks of a comparably large scope both on the individual and on the cultural level are the concepts of “world picture” and “style”.

5.1 Systematicity and Grammatical Propositions

While the belief in a universal system such as the *Tractatus* was shattered, there is still systematicity in Wittgenstein’s philosophy after 1930 insofar as the various perspectives and forms of representations he now suggests are themselves systematic; though none of them can claim universality. It may seem as if this systematicity disappeared in the later Wittgenstein as the frameworks become more and more liberal, less rigid in how they determine possibilities. But the features that make them systematic largely stay in their place. In the middle period, these are often presented in a very apodictic manner which makes them appear in stark contrast to the rather questioning tone of the *Investigations*. The position of the *Big Typescript* corpus is: Only as a possibility in a system does a word have meaning and a proposition sense; only in contrast to other possibilities can the *one* possibility which is represented become a significant sign:

> All I can do in language is to say *something*: say the *one* thing. (Say the one thing in the space of what I could have said) (MS110, 1)
>
> The grammar describes the system, the space, in which the symbol (sign) points to one position. (MS110, 126)

Importantly, these statements are expressions of Wittgenstein’s method; he does not make a thesis about signs which could be disproven if one discovered a sign without system. He rather suggests understanding those signs, *Satzzeichen,* which seem to be unsystematic insofar as they are not constituted by their place in a framework of alternatives, as a *different kind of proposition*, namely as “grammatical propositions”. To prevent another misunderstanding: it is important to see that these differences are not “big differences” in the sense of a metaphysical gap nor “small differences” between things that are in principle similar (cf. TS213, 643r); instead, they are differences in grammatical role. “The essence of metaphysics”, Wittgenstein writes, is “that the difference between factual (*sachlichen*) and conceptual investigations is not clear to it” (MS134, 153).

Statements that do not have an opposite are suspicious for Wittgenstein, especially if they are treated as a form of metaphysical truth which appears to be necessarily true because no alternative is imaginable. Rather he would consider those statements grammatical; they

belong to the geometry of our grammatical space, not to locations within them. Again the section headings of the *Big Typescript* can be quite instructive: "A proposition is a sign within a system of signs. It is a combination of signs among several possible ones and in contrast to the other possible ones". Each proposition of a language has sense only in opposition to other configurations of words in the same language. Under that heading Wittgenstein writes:

> If a proposition was not *one* possible combination among others, it would not have any function.
>
> That is, if a proposition was not the result of a decision, it would have nothing to say. [...]
>
> $\rightarrow$ as opposed to $\uparrow$ is a different sign than $\rightarrow$ as opposed to $\longrightarrow$. (TS213, 93)

While the differences between the two arrows in each pair are differences *within* a space, the difference between the pairs is one *between* different spaces, between "direction space" and "length space". The next sub-chapter then speaks of the third difference I mentioned. It is titled "Being able to imagine 'what it would be like' as a criterion for a proposition to have sense" and it highlights the point that in the sense in which I cannot imagine the opposite of a seemingly necessary statement, I can also not imagine "how it can be *like this*". Here we speak of the difference between ordinary propositions, which can always be imagined, just as well as their opposites, and grammatical propositions which seem to express a logical necessity or impossibility (TS213, 95–97;[117] cf. also PI §251).

The idea that propositions *within* a space always have an opposite, whereas grammatical, geometrical propositions do not, is a constant in Wittgenstein's thinking.[118] We have seen in chapter 3 how this was part of his motivation to employ the metaphor of space in the first place: any meaningful proposition should have a negation, a complementary region in logical space. In the middle period, the same idea is applied to the more flexible grammatical spaces. The issue comes up in the context of time and the flux and indeterminacy of experience which was responsible for the abandonment of phenomenological language.

> [...] in stating our puzzles about the fleeting nature of all phenomena, we are using the words "flux" and "vagueness" wrongly, in a typical metaphysical way, namely without an antithesis: whereas in their correct and everyday use

117. The next sub-chapter is titled accordingly: "'Logical possibility and impossibility'. – The picture of 'being able to' applied ultraphysically" (TS213, 98).

118. See also Jacquette (2010) where the connection of the proposition's bipolarity to completeness and to the different kinds of "existence" is made which we have encountered in chapter 3. By looking at §50 of the *Investigations*, Jacquette points out that the role of the prototype metre in the game of measuring is indeed beyond existence and non-existence insofar the "Ur-Meter" first of all constitutes what a metre is.

> vagueness is opposed to clearness, flux to stability, inaccuracy to accuracy, and *problem* to *solution*. (BB, 46)[119]

This passage is related to a group of remarks that is worth tracing in the *Nachlass*. Its last version has become famous as a programmatic statement of the *Investigations*: "What *we* do is to bring the words back from their metaphysical to their everyday use" (PI, §116). Being considered one of the central remarks of the so-called philosophy chapter, it has received some attention in the literature and its genesis has been tracked back to the manuscripts from 1931 (cf. Stern 1991; Baker 2004). Gordon Baker points out that the opposition of metaphysical and everyday is crucial here. He shows, that "everyday" is not necessarily tied to so-called "ordinary language", but should be understood as the antithesis to metaphysical: it could perhaps simply be substituted with un-metaphysical. He distinguishes four characteristics for the use of "metaphysical" in Wittgenstein's *Nachlass* all of which can be put in words that are based on my conception of space (cf. Baker 2004, 97–100):[120]

i) the implication of *necessity*	Necessity in my interpretation lies in the *geometry* of a space, in the framework that constitutes possibility, whereas facts are to be found *within* the space and are merely contingent.
ii) the use *without antithesis*	The geometrical or grammatical propositions are precisely those that do not have an antithesis while every region *in* space has a complementary region to it as described already in the pre-*Tractatus* notebooks. Colour exclusion can again serve as an example: that red and green cannot be at the same place at the same time is a grammatical proposition and the opposite is inconceivable in colour space.
iii) the treatment of *philosophical problems* as if they were *scientific*	If this statement ("that red and green. . . .") is uttered in the form of a scientific discovery it is a form of metaphysical nonsense. Uttered as a grammatical proposition it does not do much harm, but does not say anything positive either; it is just a description of the form of representation.

119. This refers to Wittgenstein's famous view that philosophical problems are not solved, but dissolved. They are not problems for which there are solutions, like engineering problems, but problems that arise out of misunderstandings about the way our language works. One has to re-think the question instead of looking for an answer that doesn't exist. It is telling that Wittgenstein here considers the solution as the problem's antithesis, not its complement, a kind of non-problem, not the filling of a gap.

120. This is surprising since Baker is in fact known for his opposition to the "grammatical" reading of Wittgenstein which highlights the systematicity; instead he supports a "therapeutic" reading which puts more emphasis on the variety of methods which are to be applied locally. The disagreement is largely a matter of emphasis with regards to the middle period (cf. Engelmann 2013b, 290).

iv) the attempt to get at the *essence* of a thing or a concept.	These geometrical, grammatical statements do not tell us anything about the essence of colours, as little as the statement that everything is in flux tells us anything about the nature of time – other spaces with other grammars are conceivable in which colours might be superimposed or time might be conceived as a spatialised ordering (think of the film projector or notions of a four-dimensional "block universe").

Among these especially iii) and iv) are important because they distinguish Wittgenstein's method to highlight aspects of grammar, of the geometry of our grammatical space, from what he dismisses as "metaphysics". What distinguishes them is the awareness of the form of representation, of the geometry which allows for certain possibilities and excludes others. Wittgenstein's aim is to demystify the *a priori* necessity which seems to be expressed in those statements, but which actually is only a proposition about the geometry of the grammatical space we are operating in: "A proposition *a priori* arises when a proposition about the kind of representation is clothed in the form of a statement about the represented objects" (MS157b, 3v). Baker tracks remark §116 back to the *Big Typescript* where it is set in a context that puts emphasis on *possibilities* (2004, 101). Its manuscript source stems from 1931. Therein Wittgenstein scrutinises the idea that "all is in flux", that the fleeting reality cannot be captured in language. As later in the *Blue Book* (see quote above) he points out that "fleeting" normally is used in opposition to "stable" and that, in this sense, there is nothing metaphysical about it:

> That all is in flux seems to prevent us from expressing the truth, for it is as though we can't get hold of it, since it slips away from us.
>
> But it doesn't prevent us from expressing something. – We know what it means to want to get hold of something fleeting in a description. That happens, say, when we forget the one while we want to describe the other. But that's not what we are dealing with here. And that's how the word "fleeting" is to be applied.
>
> We lead words back from their metaphysical use to their correct use in language.
>
> The man who said that one couldn't step twice into the same river said something false. One can step twice into the same river. (MS110, 33 f.)

and in the *Zwischenfassung*, the intermediate version of the *Investigations* from 1937:

> (The man who said that one couldn't step twice into the same river said something false; one *can* step twice into the same river. – And an object sometimes ceases

> to exist if I cease to see it, and sometimes not. – And we *do know* sometimes what colour someone else sees, and sometimes not.) (TS220, §111)

Baker comments instructively:

> Here the expressions 'sometimes' and 'sometimes not' evidently function as modal terms, understood as contradicting necessities expressed by using 'never' and 'always'. This parenthetical remark is evidently meant to give some examples of the (dis)solution of philosophical difficulties by bringing words back from their metaphysical to their everyday use (alltägliche Verwendung), and the crucial move is to refrain from affirming apparent necessities and impossibilities by acknowledging a wider range of possibilities. (Baker 2004, 102)

I agree with this reading. The important point is that the "wider range of possibilities" is not concerned with hidden possibilities in the same space, but with the possibility of different spaces, of looking at things differently. Another occasion where one could answer with 'sometimes' or 'it depends' is the example from the *Investigations* where "a boy racked his brains over the question whether the verb 'to sleep', for example, meant something active or passive" which is itself an example to illustrate the context-dependence of simplicity (PI, §47).

To conclude this discussion, Wittgenstein does not dismiss systems altogether after the *Tractatus*, but replaces the one comprehensive order with a more local systematicity of constitutive frameworks in which something has sense in virtue of being part of a system of possibilities. Unlike Baker and Hacker (before they split over the grammar/therapy-issue), I do not believe that the propositional systems (*Satzsysteme*), which Wittgenstein studied in 1929 and 1930, were largely abandoned (Baker and Hacker 2009, 44).[121] Instead, the systematicity of these systems, which coincide with the space concepts of that period, is reflected in a number of related concepts which I shall now turn to.

5.2 Concepts Complementary to Space

5.2.1 Logical Form and Forms of Representation

The plurality of systems that comes up in the middle period and is later emphasised in the *Investigations* is directed against logical universalism which was implied in the *Tractatus* by

121. Baker and Hackler do admit that some aspects of this view are permanent, but those listed by them are, I believe, the wrong ones: the autonomy of grammar, for example, is not a permanent feature of Wittgenstein's thinking as both Engelmann and Kuusela have shown: it neglects the instrumental use of language as e.g. acknowledged in PI §492.

the general form of the proposition and the unifying account of words whose sole purpose it is to represent objects.[122] Moreover, it is meant to highlight that this multiplicity of forms is not fixed, given once for all, but changes. An important concept to express this multiplicity of systems is what Wittgenstein calls "form of representation" or "form of expression". These are much more casual and flexible concepts than the very technical "logical form", but they are still *forms*. They are forms in the sense in which Wittgenstein introduces the word in the *Tractatus*, that is, as something that is intimately connected to the idea of logical space: form is the possibility of structure (cf. chapter 3).[123]

In 1929, Wittgenstein published a paper that is often discussed as a central piece of his middle period because it has the form of a conventional academic publication: it is a paper that was supposed to be given at a conference and published in the proceedings. Far from being a key text to the middle Wittgenstein, it merely sums up some of the developments from 1929. Especially the arguments on colour exclusion and statements of degree are discussed; I shall not repeat them here (cf. section 4.3). Anyway, Wittgenstein did not give that paper because in the time from April, when he supposedly wrote it, to July, when he spoke at the conference, he had moved on to different topics, mainly generality and infinity in mathematics as can be reconstructed from the manuscripts from that time.[124] While the paper itself contains little that is not in the manuscripts, I want to emphasise one feature about it, namely its title. Although it is, like most of Wittgenstein's titles, very general, "Some Remarks on Logical Form" is significant because *logical form* is precisely the feature of the Tractarian system that shall undergo the most profound changes. The idea is still about possibility of structures, but through its treatment of colour exclusion the paper shows that not all impossibilities are excluded by logical form alone. And that hence the investigation must be "in a certain sense *a posteriori*" (SRLF, 163). For the first time, the idea is articulated that logic might not be completely detached from experience, if only "in a certain sense"; – Wittgenstein's later philosophy is indeed marked by the tension expressed in this vague formulation: on the one hand the temptation to think that logic is empirical, on the other hand the conviction that this would abolish logic and cannot be the solution; the *role* of

122. And it is important that in his notebooks from Norway in 1937, where much of the *Investigations'* "philosophy chapter" is sketched, Wittgenstein reconstructs and dismantles the idea that there could not be any *kinds* in logic. For if there were kinds, new kinds could empirically be discovered; the claim for apriority implies a certain unity. "All numbers in logic must be capable of justification" (TLP, 5.453; cf. MS157, 47r; see also section 3.3).

123. Of course, there are also more deflationary instances of "Form" in Wittgenstein's writings and sometimes it simply means "shape", but there is a significant number of remarks in which it has the technical meaning that is connected to "space".

124. Martin Pilch has approximately dated the MS-volumes 105–107 using all dates that are available and assuming a steady writing pace for the undated passages. That Wittgenstein spoke about mathematics instead of presenting his paper is proven by a letter to Russell (July 1929).

logical propositions is still different from empirical ones (cf. section 6.2). While in 1929 he suggests engaging in phenomenological analysis, he increasingly understands propositions that are not "logical" in a straightforward sense, but still closer to logic than to experience, as grammatical.[125] In the manuscripts 1929–1932 in which the notions of grammar and space are most important, there are some significant occurrences of "form" in the sense that interests me: as a descendent of "logical form", but stripped of its Tractarian claim for universality.

In the context of the discussions of a presentist account of time (film strip and projector), Wittgenstein notes that what he is studying is actually "the possibility of movement. Hence the logical form of movement" (MS108, 27). And the idea of a phenomenological language, of an immediate description of sense data, which may lead to presentism or solipsism ("only *my present experience* is real"), is characterised in these remarks as "only *one* form of *description*, not the only possible or the only correct one as one might think" (MS108, 45; cf. also MS110, 287). It is possible, he writes, to describe the logical form of a certain domain of language by making conventions; these could be different, but changing them would change the whole way words are used (cf. MS108, 98). These conventions are like norms, stipulated by us and then misleadingly considered as something which is given *a priori* – "It is a given form of representation" (MS110, 245).

The more the focus of Wittgenstein's attention shifts towards the *use* of language, the more pressing becomes the question of the role of grammar between experience and logic. The above sketched typology of differences is nicely captured in the following remark in which the second type, that is, differences between grammars, is expressed in terms of a difference of "forms". The third type, the difference between empirical and grammatical differences is hinted at in the last paragraph:

> All one actually needs to know in philosophy is that each difference in the use of a word is a logical difference, and that we are hence dealing with different forms (whose *grammatical relationship* is at most indicated by the same word)
>
> That is, one must not pass over a difference of forms – as one may well pass over a difference of chairs when it is very small.
>
> In a certain sense there are for us – namely in grammar – no small differences. And indeed the word "difference" means something completely different than when we are dealing with the difference between two things (MS111, 56–57).

In the notebooks leading to the *Investigations,* Wittgenstein noted that the *a priori* should become *one* form of representation, not *the* form (cf. MS157b, 3v), the one form which so

125. "Phenomenology is Grammar" (TS213, 437).

greatly impressed him in the *Tractatus*.[126] In the final version, this view finds its way into the discussion of the ability to read: a naturalistic interpretation of reading which seems to be *a priori* necessary is here characterised as "a form of representation which is very appealing to us" (PI, §158). However, it is also one example of how "our forms of expression, which send us in pursuit of chimeras, prevent us in all sorts of ways from seeing that nothing extraordinary is involved." (§94). And in the manuscript that links the corpus of the *Big Typescript* with the *Investigations*:

> The "great", difficult, problems of philosophy are what they are not due to an unheard of, subtle, and mysterious state of affairs which we should examine, but due to the crossing of a large amount of misleading forms of expression at this place. (MS116, 217–218)[127]

It is an essential part of Wittgenstein's philosophy to criticise these misleading forms and – as he also said about creativity in mathematics – invent new ones.[128] "The philosophical problem appears to be unsolvable. Until one sees that there is a disease of the form of representation" (MS115, 110). And in the philosophy chapter, Wittgenstein describes (part of) his grammatical method as the substitution of one form of expression with another (cf. §90). All these uses of form share with "space" their role as a framework which determines a certain perspective on an issue and on what is considered possible. If this perspective causes confusion, the space must be clarified or new spaces be invented.

5.2.2 Notation, Symbolism (Mythology)

It has been pointed out that in the *Tractatus* Wittgenstein is introducing a notation. Recently Kuusela advanced a compelling interpretation pointing out the many phrasings that sound like the introduction of notational concepts („I will call...") and thereby solving the so-called paradox of the *Tractatus*, namely that its own propositions are nonsense. By interpreting them as introductory, Kuusela succeeds in giving an account of how they can have a function despite being nonsensical[129] (cf. Kuusela 2019a). What he does not address specifically is

126. On the picture theory and the idea that there must be a logical form, which accounts for all possibilities world and language have in common, he comments retrospectively: "Thus, a certain generalisation intrudes itself upon me, a certain form of representation, a certain aspect" (MS116, 123).

127. In the *Investigations*, the tension between meaning as use and understanding "at a stroke", which I shall discuss in the next chapter, is characterised as such a crossing (cf. §191).

128. With regards to mathematics as inventing "new forms of representation", see MS119, 95. On different "forms of questions" in mathematics, see also MS112, 130.

129. This is the notorious problem of the resolute reading. It claims to take seriously the remark that the propositions of the *Tractatus* are nonsense and refuses to distinguish different kinds of nonsense; but if this is true, why should exactly these nonsensical propositions help us to get to the right point of view?

the role of notations in Wittgenstein's later philosophy, but he does mention one remark about his discussions with Ramsey in 1929. Wittgenstein complained about his friend's neglect of the importance of a specific form and highlights his own preoccupation with notations:

> R[amsey] does not comprehend the value I place on a particular notation any more than the value I place on a particular word because he does not see that in it an entire way of looking at the object is expressed; the angle from which I now regard the matter. The notation is the last expression of a philosophical view (MS105, 10).

I think it is significant that Wittgenstein insists on the importance of notations in that period in which he is also obsessed with spaces and calculi. (especially in MS106 and MS113, cf. section 5.3.1). In 1930, in the remark already quoted in chapter 4, Wittgenstein reflects on his own frequent and various usage of "space" in his own writings. „Our notation or mode of expression", he writes, constitutes the way we look at things which is expressed in this space-terminology. The remark implies that the "spatial" mode of expression is a notation, or rather: many notations. For he also admits that he uses "spaces" in various ways: sometimes he wants to highlight an analogy with physical or geometrical space, sometimes he wants to point out differences in a more metaphorical sense between different spaces (cf. MS108, 135).

What the concept of "notation" implies is that constitutive frameworks can be *constructed*. However, even though there are those expressions in the *Tractatus* in which this constructivist aspect is visible, as Kuusela has shown, this aspect is easily overlooked due to the universality of the Tractarian system. It makes it seem as though *the* right notation had simply been discovered, *the* logical space simply been described: the truth of these thoughts is "unassailable and definitive". In 1932, by contrast, Wittgenstein is more aware of the possibility to invent spaces (like the mathematician, cf. section 4.2) and to construct new notations „I offer the confused person a rule and he accepts it. One could also say: I offer him a notation" (MS113, 27r).

> The description of a new, for example a more perspicuous, notation (for perspicuity is central for us) is of the same kind as the description of one of those languages which children invent or learn from one another in which, e.g. each vowel of normal words is doubled and between each part of the doubling a "b" is inserted. Here we have come quite close to the concept of game [...]
>
> What seems to lead us astray here is the double meaning of the word "description": when one sometimes speaks about the description of a real house or tree

> etc., and sometimes about the description of a form (*Gestalt*), construction etc., of a notation, of a game. (MS113, 28v)

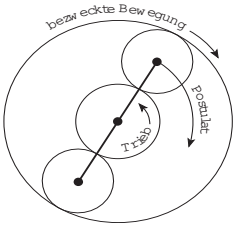
In the last paragraph, the distinction between experience and grammar is framed as the problem of the ambiguity of "description". Indeed, Wittgenstein's repeated claim to be purely descriptive can be puzzling due to this double meaning. It is connected to the different kinds of difference which I distinguished above: while the description of houses, trees and so on belongs to the sphere of experience, the description of notations and games belongs to that of grammar. The former corresponds to the differences between trees and houses, the latter to those between different grammatical spaces. The "double meaning" of "description" corresponds to the ambiguity of "difference", that is, to the difference between empirical differences and grammatical differences.

Even though the constructivist element (more on this in section 6.2) is quite visible in the context of notations, there is still the temptation, which Wittgenstein had succumbed to in the *Tractatus*, to give a "mythology of the particular notation or symbolism" one is using (MS108, 104). What does mythology mean here? It means to conceive of the second type of "description", of a description of grammar, as if it were a *de re* description of the first type, to hypostatise the notation as an underlying structure of reality instead of seeing it as a formal aspect of the way we describe things. To work against this kind of mythology, to work "against the myth-building tendencies in our mind" (MS158, 28r) is a central aspect of Wittgenstein's understanding of philosophy: "One could say philosophy purges our thinking from a (wrong /misleading/) mythology" (MS109, 210–211).[130] Until shortly before his death, he conceived of the constitutive structures that frame our approach towards things as mythologies; in his latest writings he will explicitly emphasise the fact that empirical propositions, too, can have the role of such "mythological" statements, but also that the mythology can change (cf. OC, §95; 97).

5.2.3 Hypotheses, Theory, Image

Other concepts in the orbit of "spaces" are the more scientific concepts of hypotheses and theories. Of course, hypotheses are the paradigmatic example of sentences that may be true or false. But Wittgenstein acknowledges the possibility to turn them into postulates which will be upheld no matter what:

130. In this remark, Wittgenstein also makes a reference to Paul Ernst's epilogue to Grimm's *Tales* (1910) which seems to have influenced his use of the word "mythology". It is another example of creative appropriation. For Ernst, mythology is indeed connected to misunderstanding "the logic of our language", but has rather positive connotations as a source of poetic expression. Wittgenstein instead criticises similar misunderstandings and mythologies in the would-be scientific conceptions of his contemporary philosophers.

> One can compare a part of a hypothesis to the movement of a part of a gear, a movement which one can stipulate without predetermining the envisaged movement. However, one must then set up the rest of the gear in such a way that it results in the desired movement. I am thinking of a differential gear.
>
> 
>
>
> Once I have made the decision that a certain part of a hypothesis is not to be altered whatever the experience may be that I want to describe, then I have stipulated a manner of representation and that part of the hypothesis is now a postulate. A postulate must be such that it cannot be refuted by any conceivable experience, even if it may be troublesome to hold on to the postulate. (MS108, 109)[131]

Later, he uses the same analogy to show the same relation between an empirical hypothesis to an *a priori* postulate and frames it in terms of the already mentioned "form of expression":

> How does the assumption of a fact (*Sachverhalt*) turn into an assumption of a form of expression? Picture of the differential gear. (MS116, 233)

This view is basically similar to Poincaré's idea of hypotheses which may be elevated to conventions and then form a quasi *a priori* system of accepted beliefs.[132] A number of these postulates may be turned into a theory which, of course, has a larger scope. Hertz's *images*, for example, are theories to account for the totality of mechanics. They can be more or less *correct* and are insofar revisable, but it is also clear that within such a system it makes no sense to question the principles themselves. As he makes clear in the beginning, Hertz holds that his principles are *a priori* in a Kantian sense (Hertz 1899, 1) and that the title of his book is motivated by the aprioric status of his "principles" (they are, however, testable for their permissibility, correctness and appropriateness, but this is a different testing than testing a hypothesis within a theory). Wittgenstein's general opposition towards theories in philosophy[133] is in one sense directed against the universal claims they might suggest (cf. Kuusela 2008); in another sense a theory is a scientific doctrine, something that can

131. The schematised image is taken from the *Vienna Edition*, vol. 2, p. 230.

132. This is the reason why Carnap justified himself in the famous plagiarism affair by referring to Poincaré: Wittgenstein had accused him of having stolen the notion of hypotheses which can never be completely verified, but only "stand the test" and then eventually be turned into conventions. This idea indeed occurs in Wittgenstein's remarks from 1929/30 and in Carnap's paper *Die Physikalische Sprache als Universalsprache der Wissenschaft* (1931). Wittgenstein claimed that Poincaré could not have known his idea because he did not have Wittgenstein's account of propositions and grammar. But it is understandable that Carnap saw them as reasonably similar (cf. Wittgenstein's letter to Schlick 8/8/1932).

133. Wittgenstein's rejection of theories is much discussed in the literature, see for example Kuusela 2008; Hacker 2012b; Engelmann 2011; Uffelmann 2018 etc. All accept this rejection. Some scholars (Kuusela, Engelmann), however, believe that other scholars (Hacker, Glock) implicitly impute a theory to Wittgenstein by

empirically be refuted and hence not what Wittgenstein was interested in (cf. PI, §109; cf. Hacker 2012b). The philosophy of the *Tractatus* was a theory in the first sense: Wittgenstein compares his former assumption that *all* propositions must be pictures to Freud's theory that *all* dreams are wish-fullfilments (cf. MS157a, 55r–55v). It was not a theory in the second sense: already the *Tractatus* was concerned with "what *precedes* every experience" (TLP, 5.552; see also Kuusela 2019a).

An interesting crossed-out remark from the manuscript of the *Investigations* contains an addendum to §3 which insists that Augustine's system of language is only valid for a narrowly circumscribed region. In the cancelled sentence, Wittgenstein asks the reader to compare this with the „theories of macroeconomists" (TS227, §3). First, it is interesting that Wittgenstein brings up the notion of theories in this context which can be explained by means of the conceptual proximity of linguistic systems and theories which I have pointed out (both are spaces, games etc.). Second, it is interesting that he chooses such a questionable „science" as macroeconomics in which we are probably less tempted to say they are „true", but rather appropriate in their domain.[134] This might also be one of the ways in which Piero Sraffa influenced Wittgenstein's thinking beyond the famous legend of the Neapolitan gesture.[135] The remark, although crossed out, indicates that it is not completely absurd to conceive of theories in terms of spaces or games. I repeat that Wittgenstein rejects theories insofar as they are concerned with scientific discoveries as opposed to conceptual clarifications; he also rejects them as frameworks that claim universality.

However, a theory as a non-universal framework, constructed in order to clarify a specific situation, would in principle not be in conflict with his philosophy. Like a word only acquires meaning within a space (or a language-game, as we will see), he argues that "something is an experiment only in the space of a theory" (MS117, 97). Remember also Wittgenstein's positive references to relativity theory throughout all of his writings from

absolutising his method of philosophy: that one should look at language under the aspect of rules, grammar or language-games is itself a theory, they claim, and should therefore not be taken as Wittgenstein's dogmatic position. I shall not dwell too much on this dispute. I argue that "meaning is use" stands in some continuity to "meaning: the place of a word in grammatical space" and acknowledge that Wittgenstein said about this method that it works for "a *large* class of cases [...] – though not for *all*" (PI, §43).

134. And it is perhaps today as relevant as it was at the time when we think about the consequences economists derive from simplified modells up to this day.

135. Michael Nedo told me in private conversation that Sraffa – somewhat tongue-in-cheek – did not remember that incident, but that he had heard the story so often by other people that it must be true. Engelmann gives a more comprehensive proposal of what Sraffa's influence consists in, namely in making Wittgenstein aware of the importance of use as opposed to an autonomous grammar. While I agree that Sraffa must have pointed out the plurality of frameworks one has to take into account and that these are constituted by empirical findings and not completely autonomous, I do not agree with Engelmann's framing which presents Sraffa as convincing Wittgenstein to drop grammar completely and to adopt the "anthropological view" (cf. Engelmann 2013a).

1930–50.[136] Elsewhere, he uses theory and language-game quasi synonymously when he compares "the position of a picture in a theory, in a language-game" to the location of a mathematical expression in an equation: the expression alone does not tell us anything without its environment (cf. MS120, 50v). And in 1945, after the completion of the preface of the *Investigations*, he rejects "philosophical theory" in terms of his distinction of *describing* and *explaining*. While a theory tries to explain things, philosophy ought only to describe. He continues that what makes the task of describing so difficult is that we see reality, as it were, through a net which makes us confuse features of reality with features of the net. What is very subtle about this remark is that it seems to simply confirm the anti-theoretical gist Wittgenstein is famous for, but then goes on as follows: "a completely unordered description is of no value for us. But to see the relevant structure is difficult because it is hidden by the grammatical net" (MS130, 217–19).[137] This is somewhat puzzling and seems to indicate Wittgenstein's shifting terminology. He certainly rejects a "philosophical theory" as he reinforces in the preceding remark, but he also rejects the idea that reality can be described without any "net", without any "order". While this remark sounds as if there was a reality that can be described independently (cf. also PI §103),[138] I can only make sense of it if it means: we have to provide a *different* net, a different order, which is "relevant" insofar as it solves a particular confusion that has been prompted by the misleading grammatical net of our ordinary and established language. In this sense, it could even be a philosophical act to suggest a new theory, only not in any universalist sense:

> We want to establish an order in our knowledge of the use of language: an order for a particular purpose, one out of many possible orders, not *the* order. For this purpose we shall again and again *emphasize* distinctions which our ordinary forms of language easily make us overlook. (PI, §132a)

Nor in the sense of a theory which captures reality better than our ordinary language. Rather, these new theories should only be put forward to clarify shortcomings or confusions in the existing ones.

> This may make it appear as if we saw it as our task to reform language.

136. Einstein, too, initially insisted that his theory was not epistemologically *necessary* but a more promising and satisfying way to look at things (cf. Einstein's letter to Schlick from 6th September 1917: quoted in Hentschel 1986, 481).

137. The word "relevant" is marked as questionable with a wiggly underline and for "the relevant order" there is a textual variant "one relevant order". Both modifications clearly aim to prevent any essentialist and absolutist connotations about *the* relevant order.

138. In an earlier version of §103, he seems to consider that the analogy of the glasses through which we see everything can be misleading as it suggests that we could also see without any glasses (cf. MS157b, 3). Strangely, Wittgenstein also says that the glasses in his simile could not be exchanged either. Maybe he has in mind the great power a certain ideal can have over our minds.

> Such a reform for particular practical purposes, an improvement in our terminology designed to prevent misunderstandings in practice, may well be possible. But these are not the cases we are dealing with. The confusions which occupy us arise when language is, as it were, idling, not when it is doing work. (PI, §132a–b)

The proposal of a theory in a positive sense would be, again, very similar to Hertz's *images* which were supposed to make disappear certain questions that cannot be answered in a theory that suggests contradictory or confusing connections.[139] Philosophy would then consist not in describing reality as it "really" is, without any net, but in suggesting different nets in order to solve problems that arise out of the existing one which is wrongly taken as *the* correct one. If the suggested theory does not solve the problem, Wittgenstein is happy to drop it and suggest another one, an offer which has frequently been quoted in order to illustrate Wittgenstein's non-dogmatism (cf. LFM, 22; Engelmann 2011, 87). In 1946, when discussing psychology and psychological theories like that of William James, Wittgenstein seems to use "theory" in the more positive sense I want to highlight, giving further evidence of the connection between theory and notation:

> The theory says really: 'it could be like *this* … '. And the purpose of the theory is that it illustrates a concept.
>
> But it can illustrate it better or worse; more or less to the point. The theory is, so to speak, a notation for a psychological phenomenon. (MS132, 48)

Even though he rarely calls frameworks in this positive sense "theories", I think it would be possible in the qualified sense sketched in the preceding paragraphs. Such a theory would not claim to have discovered anything true by means of which something can now be explained, but it is, like a new notation, rather the description of a new way of looking at things: "the real merit of Copernicus or Darwin was not the discovery of a true theory, but of a fruitful new aspect" (MS112,117). The word "theory" he seems to reserve for the negative cases in which the wrong framework, which is taken to be universally valid, hinders us to see that things could be different (and would be less confusing). Wittgenstein repeatedly points out

139. Indeed Wittgenstein also considered using a line from Hertz's preface to the *Principles of Mechanics* as a motto (cf. Baker and Hacker 2005, II, 30). I quote it with a bit more context, although Wittgenstein wanted to use only the last sentence "Our confused wish finds expression in a confused question as to the nature of force and electricity. But the answer which we want is not really an answer to this question. It is not by finding out more and fresh relations and connections that it can be answered; but by removing the contradictions between those already known, and thus perhaps by reducing their number. When these painful contradictions are removed, the question as to the nature of force will not have been answered; but our minds, no longer vexed, will cease to ask illegitimate questions" (Hertz 1899, 7–8).

how deeply rooted in our minds some misleading forms of expression are[140] and attributes the need for such forms to "the power of theory" (MS131, 96).

The positive account of "theory" may also shed some light on Wittgenstein's famous criticism of Ramsey as a "bourgeois thinker". Following the earlier remark on Ramsey not seeing the importance of notation, one can interpret Wittgenstein's problem with Ramsey's attitude as follows: In chapter 4, I compared Wittgenstein's ideas about creativity in inventing spaces with Kuhn's revolutionary science which introduces a new paradigm. Paradigm may be a wider concept than theory, but it captures the positive aspects of theories I am highlighting. Thus, "bourgeois" can be read as opposed to "revolutionary" in the Kuhnian sense: as a sort of puzzle-solving activity which may involve a considerable degree of skill, but is still distinguished from the creativity that is required for paradigm change. Ramsey would in this sense be bourgeois because he prefers to work within an unquestioned framework, not thinking about its notation or form. Whether this criticism is fair, I do not want to judge, but it seems very likely to me that these remarks are connected with the idea of constitutive frameworks rather than with political attitudes.[141] There are some indications supporting this view. The political imagery – Ramsey was only concerned with a "given community", with "the improvement of *this* state" while the "possibility of other states" unsettled him – is meant to distinguish the two philosophical styles: for Wittgenstein, "the philosopher is not a member of a community of ideas (*Denkgemeinde*)", but instead engages in revolutionary thinking by offering different ways of looking at things (cf. MS112, 70v and 72r).

5.3 Frameworks Replacing Spaces

I shall now turn to concepts which are more frequent than those discussed so far. They are used heavily in certain periods of Wittgenstein's writing and it is striking that they emerge when the literal occurrences of "space" decrease. This is an indication that, unlike the discussed concepts, which complement the space idea, the concepts of calculi and games rather substitute spatial frameworks. Even though they take up many functions that the former were supposed to fulfil calculi and games differ from spaces in a substantial way:

140. "The forces that determine *how* something is represented are as great as those that insist on the truth of what is represented" (MS131, 180).

141. For such a view see Gakis 2018, 245. However, this accusation would be strange: although Wittgenstein had some connections to socialist movements in Cambridge and beyond, Ramsey was no less active in these circles and probably even more committed. And Wittgenstein himself had obviously the most bourgeois background one can imagine.

while the latter are static frameworks, the former are more dynamic. This is at first a marginal difference, but becomes increasingly important as I shall show in section 6.1.

A rough idea of the relation between space, calculus, and game can be attained by considering the frequencies of the terms in the manuscript volumes from 1929 to 1932. The significance of such a quantitative comparison is of course limited, especially as the table does not account for related terms such as "Geometrie" or adjectives such as "räumlich" and "geometrisch". Moreover, there are "jumps" between the manuscripts, for example when the remarks from MS106 are continued on the verso pages of MS105 which had been left free when Wittgenstein started writing manuscripts in February 1929. Hence, an occurrence of "game" in MS105 can in fact be later than an occurrence of "space" from MS106. What is more informative are the year dates and the ratio between the three frequencies in each volume. Over the course of the four years a general tendency becomes evident.[142]

MS-Volume	pages	year	"Raum"	"Kalkül"	"Spiel"
105	134	1929	30	1	1
106	298	1929	46	1	3
107	300	1929-30	45	3	7
108	300	1929-30	59	8	10
109	300	1930-31	14	3	19
110	300	1930-31	34	19	18
111	200	1931	16	71	33
112	270	1931	15	43	72
113	270	1931-32	32	93	42
114	60 (290)	1932 (33)	9 (16)	7 (43)	5 (71)

While I argued against a direct connection between logical space and Boltzmann's phase-space, in light of these numbers one may speak of a "space-phase" after 1929 in which "space" is Wittgenstein's preferred concept to account for the grammatical insights he wants to highlight. The occurrences of "space" decrease slightly after their peak in 1930 which goes along with a significant increase in occurrences of "game" – clearly an indicator of transition in Wittgenstein's thinking in light of the further use of these concepts: "space" disappears almost completely after 1933, whereas it is obvious how important "game" is in the later philosophy of the *Investigations* and *On Certainty*. What is particularly interesting, though,

142. The data in this table is taken from *wittfind*, a web-application developed by the universities of Munich and Bergen in order to make Wittgenstein's *Nachlass* digitally searchable. (wittfind.cis.lmu.de, access date 13/11/2018). Only parts of MS114 belong to the continuous stream of remarks which stops in June 1932. In 1933 Wittgenstein starts a re-working of the *Big Typescript* in the second part of that volume (numbers in brackets).

is the striking cluster of "calculus" in 1931–32, a concept which occurs only occasionally in later manuscripts, mostly in reflections on mathematics (e.g. MS117)[143]. To better understand these numbers it is necessary to look at the function of the words in their contexts as I have done with "space" in the last chapter.

5.3.1 Calculi

We have seen that grammar was conceived in terms of geometry. In the calculus-phase, Wittgenstein considers grammar a "pure calculus" (MS113, 62r; 128r). While the meaning of a word was defined as its "place in grammatical space" and similarly the sense of propositions was accounted for in spatial terms, now the "proposition has its content as an element of a calculus" (MS111, 183; cf. MS114, 43r); and: "what role the proposition plays in the calculus, that is its sense" (MS113, 42r).

Analogies to space were used in the context of intentionality in order to account for the idea that thought and world, expectation and fulfilment, are both phenomena of language. Agreement between thought and reality relies on agreement in grammar, that is, on their being located in the same grammatical space. In 1931, Wittgenstein writes instead: "When the words 'agreement with reality' are used, they are not used as a metalogical expression, but as part of a calculus, as part of ordinary language" (MS113, 49v). Generally, the word calculus is often invoked to stress the impossibility of metalogic, by which Wittgenstein understands the description of logic, of a geometry, a calculus, in a kind of neutral and maximally general language which is independent, as it were, *outside*, of the individual calculi. However, the description of the geometry of a space is already part of another space, a new calculus, not a meta-calculus.

The invention of new calculi follows the same pattern as the invention of new spaces; to calculi, too, Wittgenstein's association to the Kantian *synthetic a priori* applies which he exemplifies with the discovery – better: invention – of the space of periodicity, that is, a space that allows for the possibility of numbers with periodic decimals, and an according notation: $0.33333\ldots = 0.\dot{3}$

143. Susan Edwards-McKie argues for two strands of interest in the later Wittgenstein, one focusing on calculi and mathematics, one focusing on games and practices (in private conversation). While this has some plausibility, particularly in light of the separation of mathematical remarks during the genesis of the *Investigations* (cf. Baker and Hacker 2009, 3–19 ; cf. also Edwards-McKie 2015), I here argue for the similarity of these two strands. First, I do not want to split Wittgenstein's personality further into a mathematical and a practical Wittgenstein, as the existing fragmentation along chronological lines already causes enough trouble (cf. Kienzler 2017); second, I think there is enough textual evidence to stress the connection between calculi and games and between mathematics and the more anthropological remarks.

> And the discovery of periodicity is in reality the construction of a new sign and a new calculus (MS114, 10r).
>
> He who discovers periodicity, invents a new calculus. The question is, how is a calculus with periodic division distinguished from one which does not know periodicity? [...]

In the next remark, he exemplifies these distinctions between calculi, which are used similarly to distinctions between spaces, with the juxtaposition of a finite and an infinite space (treated here as individual signs, belonging to different calculi) which we have encountered in the last chapter:

> [...] I could, by the way, say that the sign "1, 1+1, 1+1+1, and so on" is distinguished from the sign "1, 1+1, 1+1+1" by its application. That they belong to different calculi. (MS111, 52)

That the periodic division can be continued infinitely is not a fact, but more akin to an aspect of geometry, something which determines the possibilities in space like rules of a calculus determine the possibilities of applications: "When we say: 'the possibility of producing decimal positions in the division of 1:3 is infinite', we do not state a fact of nature, but we give a rule of a calculus" (MS113, 98r). Again, distinctions *between* calculi are something else than differences *within* a calculus:

> One might perhaps say: but 'integer' is opposed to 'rational number', 'real number' etc. But this difference is a difference of the rules (of the rules of the game which are valid for them) – not of a position on the chess board – not a difference for which one could use different coordinated words in the same calculus. (MS111, 123–124)
>
> Two mathematical entities one of which I can compare with every rational number in my calculus, but not the other, – are not numbers in the same sense of the word (MS113, 134r; cf. again MS105,1)

The notion of impossibility, which was one function of logical space and is in the background of all variants of grammatical spaces – they exclude possibilities from the realm of colours, physical time, visual space etc. –, is likewise accounted for by calculi in this period:

> When one asks: does it have sense to say 'there will never be this and that'? – well, what evidence exists for this; and what follows from it? – for if there is no evidence for it – not that we have not yet been able to get them – but that it has not been arranged for by the calculus, then this determines the character of the propositions (MS113, 44r).

The proximity of these ideas to the usage of "geometry" and "space" as systems of grammatical rules as delineated in the last section is expressed in the following remarks. They highlight the somewhat inflexible rule-governedness of calculi which shall become problematic later:

> What applies to the word "language" must also apply to the expression "system of rules". Hence also to the word "calculus". (MS110, 200)

> I look at language and grammar under the aspect of calculus / *as* a calculus / under the form of the calculus, i.e. of the operations / the operating according to fixed rules / i.e. as a process according to fixed rules. (MS111, 67)

The focus on fixed rules shall return in a central remark of the *Investigations.* It has led many scholars to dismiss calculi and language games as tempting, but misleading concepts.

> F. P. Ramsey once emphasised in conversation with me that logic was a 'normative science'. I do not know exactly what he had in mind, but it was doubtless closely related to what only dawned on me later: namely, that in philosophy we often *compare* the use of words with games or calculi which have fixed rules, but cannot say that someone who is using language *must* be playing such a game. [...]
>
> All this, however, can only appear in the right light when one has attained greater clarity about the concepts of understanding, meaning and thinking. For it will then also become clear what can lead us (and did lead me) to think that if anyone utters a sentence and *means* or *understands* it he is operating a calculus according to definite rules. (§81)

However, the temptation is in any case a significant one for Wittgenstein as much of his philosophy is concerned with these kinds of framework. Second, they are not dismissed, but still regarded as useful objects of comparison. And it is hard to neglect that making comparisons is at the heart of Wittgenstein's philosophy and that it is precisely *these* objects of comparison which characterise his way of clarifying problems. Third, the remark is qualified as for calculi and games "with fixed rules". The idea that there are frameworks which are more flexible and do not feature a metaphysical necessity is one reason for the preference of games in the later period and for Wittgenstein's preoccupation with questions of necessity (games with fixed rules are in this sense more like calculi, but they allow for moves which are not rule-governed, and games are conceivable that have very flexible rules or no discernable regularity at all).

To conclude this section, the whole idea to compare language to calculi is based on the earlier comparison to spaces and geometries; it can be seen as a transition to "language-games" which is a concept close to calculus anyway. "This is clear: that the question 'what is a calculus' is of the same kind as 'what is a game' or as 'what is a rule"' (MS111, 75). Calculi are not only etymologically related to games,[144] and not only connected to them in Wittgenstein's reflections on mathematical formalism,[145] but also structurally similar in their role as frameworks of possibilities which allow for certain moves and exclude others.

5.3.2 Games

Although the centrality of games is obvious in Wittgenstein's later philosophy there have been warnings against exaggerating their importance. Pichler argues that the method of language-games proper was developed in the *Brown Book* and was changed on the way to the *Investigations*, notably in MS142. The *Brown Book* is obviously more didactic, aiming to teach something by means of language-games, whereas the *Investigations* are more open: just presenting the language-games like a play might be performed without a particular didactic aim, simply juxtaposing different views (cf. Pichler 2004). Similarly Kuusela argues against an overblown importance of games because in his view, they are simply objects of comparison (again PI §81 supporting this) and only one of various methods for treating philosophical confusions (cf. Kuusela 2008, 172–176). But it is certainly Wittgenstein's method to use *these* objects of comparisons. One could also compare language to other things, but as a matter of fact, games are a very useful tool (as is space) to highlight the kinds of difference Wittgenstein cares about. Moreover, his point is that language need not conform to a calculus or game with "fixed rules", but towards the end of this section I am going to investigate Wittgenstein's idea to speak of language-games whose rule are not fixed, but flexible and changeable over time. That the concept of games is more flexible and allows for loose and "incomplete" rules motivates their preference over calculi in the *Investigations*.[146]

While the connection between calculus and game is easy to see, that between game and space is less clear. Wittgenstein's chess analogies can help to illuminate this connection. Max Black noticed the possibility to compare Tractarian logical space to the rules of chess which constitute a space of possible positions (cf. 1964, 55). This intuition is very close to my reading of Wittgenstein's space concept, although it neglects the difference between static locations and dynamic moves that shall be central in the next chapter. Wittgenstein himself

144. In Latin, "Calculi" means little pebbles or stones which were used in ancient Rome to calculate, but also as tiles in games. (cf. Schütt 2004, 353).
145. Cf. the sub-chapter in the *Big Typescript* "Mathematics compared to a game" (TS213, 530 ff.).
146. See also Baker and Hacker 2005, I, 51.

made similar comparisons in the middle period where the "space" idea changes. In this time, the concept of "geometry" is extended to all kinds of domains, even to arithmetic, and in this sense he raises the question whether the "game of chess, too, (or any other game) is not a kind of geometry?" (MS108, 117).[147] And in his conversations with Waismann and Schlick, Wittgenstein argues that it is "the totality of the rules of a game that yields the logical position of a pawn" (WVC, 104).[148] These analogies highlight the similarity between space and game as it has already been indicated in the last chapter where the incommensurability (and yet completeness) between number spaces was compared to the difference between chess and draughts (cf. MS111, 29). The similarity is that in both cases a range of possibilities, not facts, is constituted. A geometrical object is constituted by the axioms, a word is determined by "its place in grammatical space", a piece of chess by its role in a game. "Geometrical laws constitute the cube like the rules of chess constitute the king" (MS112, 112). Outside of a space or a game a word does not have sense. In 1931, when thinking about indexicals and the impossibility to define them ostensively ("this is here" is empty without context), Wittgenstein states a typical result for his thinking in that time: "this shows that the space is missing in which this statement is to be made in order to have sense" (MS109, 246). Later, in the *Investigations*, this emphasis on context-dependence recurs in the terminology of games; to give just one example with similarly "empty" words which lack a clear conceptual framework to be used in:

> These concepts: proposition, language, thought, world, stand in line one behind the other, each equivalent to each. (But what are these words to be used for now? The language-game in which they are to be applied is missing.) (§96)[149]

Another feature space and game have in common, at least since the disintegration of logical space, is their plurality. In the first paragraphs of the *Investigations*, Wittgenstein quotes Augustine as an example of how language has often been viewed – mainly as naming objects – and imagines a language-game in which two people build a house using such a language in which words are simply coordinated with objects. We are asked to conceive of this as a "complete primitive language" (§2) and to observe that what is described by Augustine is indeed a "system of communication":

147. In the following remark, he writes something that is related to my idea about a shift from static possibility as exemplified by pictures and spaces to the question of application and interpretation: "The thought is picture and interpretation at the same time. The picture is static, but it is applied in a dynamic context".

148. In this context, Wittgenstein also speaks of the differences of geometrical and physical possibility when he states that the word "can" in "The bishop can move only diagonally" is grammatical.

149. In the language-game of §8, the word "this" does indeed have a function, but it is not the function of a *name* in the sense of a "rigid designator". It is radically context dependent. Wittgenstein is here countering the idea that "this" is the archetype of naming, establishing a direct connection between a word and an object. But it does so only in a space and could refer to any location within that space, to any move in a game (cf. §38).

> only not everything that we call language is this system.[150] And one has to say this in several cases where the question arises "Will that description do or not?" The answer is: "Yes, it will, but only for this narrowly circumscribed area, not for the whole of what you were purporting to describe."
>
> It is as if someone were to say, "Playing a game consists in moving objects about on a surface according to certain rules . . ." – and we replied: You seem to be thinking of board-games, but they are not all the games there are. You can rectify your explanation by expressly restricting it to those games. (§3)

If one regards the *Investigations* as the most finished work by Wittgenstein after the *Tractatus*, this remark can be seen as the result of the struggles ("over the last 16 years": 1929–1945) to overcome the universalism of logical space. The system of the *Tractatus*, although being much more elaborate than the primitive language-games considered here, does not achieve what it aimed for, namely to provide a universal account of language; it can be rectified by restricting its domain.[151]

At this point, it is in order to give a brief overview over the first part of the *Investigations* which on the one hand marks a deviation from the middle Wittgenstein because it follows a different, a polyphonic, strategy instead of the more straightforward attempts to find fixable insights which last up to the *Brown Book* (cf. Pichler 2004); on the other hand it shows nicely how the whole idea of spatial thinking is still present and how it is connected with games. Since I do not have the space here to reconstruct the *Investigations* in detail (as has been done by Baker and Hacker in their analytical commentary), I highlight selected passages that touch the issue of spaces and games.

The first mention of games is set in the context of language learning and not related to my concern, except for Wittgenstein's statement that he shall "also call the whole, consisting of language and the activities into which it is woven, a 'language-game"' (§7). However, considering different *kinds of word* (cf. §17) already comes closer to the idea of grammatical spaces with different rules as exemplified in the middle period, especially if we consider the examples that are given (colours, numbers, physical objects etc.) and the emphasis in the surrounding remarks on their multiple possibilities of classification depending on how they will be used.[152] Obviously, the term "kinds of word" differs in scope from, is narrower

150. Cf. in 1931: "Augustine describes indeed a calculus, only not everything we call language is this calculus" (MS111, 18).

151. This is by the way another comparison with relativity theory and its conception of space. Even in General Relativity, Euclidean Geometry has not disappeared, but become a special case of the variable structure of space, useful in small dimensions.

152. Remark §12 of the *Investigations* comes straight from the peak of the space-phase in 1930, where its aim was to show that words may seem superficially similar, but function very differently in different systems or spaces (cf. MS107, 231–232).

than, the frameworks we have encountered so far, but like dimensions of a space, they are manifolds of possibilities. Wittgenstein distinguishes in the example of someone buying "five red apples" (§1) and in the (extended) language-game of the "builders" (§8) different kinds of word which challenge the initial assumption that all words denote objects. In these games, there are number-words, colour-words, words for kinds of fruit, words for the shapes of physical bodies, and two kinds of indexical ("this", "there"). Each of these kinds is a system, or a space, insofar as one location in this space can only be addressed as opposed to the other locations in that space which would also be possible: I want apples as opposed to other kinds of fruit, red ones as opposed to yellow or green ones, and five as opposed to any other number. Likewise it belongs to the geometry of this space/game that the apples must have a colour and that there must be a certain number of apples (even zero or "no apples" would be in "apple-space" as we have seen in section 4.3).

The fact that various of these kinds of words can be used in *one* language-game already shows that the scope of such a range is narrower than that of games. Here we re-encounter the relation of the Tractarian logical space and its sub-spaces, although on a smaller scale, and of course with the difference that there is not one logical form common to all the sub-forms: their only principle of unity is that they are jointly used in a certain activity. The kinds of words may be considered as dimensions of the larger space called "the-builders-game": the dimension of shape has four locations, that of number perhaps 26,[153] the indexicals point to any location in either "shape" space ("this") or the physical playing field ("there"). The first important thing to note is that there are differences in the way these words are used, but the second is that there is no *a priori* determinable and fixed set of kinds of word which could be given independently of a purpose. How words are grouped together depends essentially on the aim of classification (cf. §17).

Again, there is a chess analogy to be made about these multiple ways of categorisation: one could, for example group chess pieces into black and white, but also into pawns and other pieces and within the latter, knight and bishop might belong to one group and rooks to another (minor and major pieces). But one could just as well group rook and bishop together and single out the knights if one wants to distinguish the straight movement of the former against the more flexible of the latter; the relative value of the pieces may also change over the course of a game and thereby also the groupings. A similar argument is used with regards to simplicity: a chess board can be naturally divided into 64 black and white squares and in this case the squares would be the simple objects which make up its structure and constitute the possibilities (possible locations for chess pieces). But Wittgenstein suggests different

153. Its symbols are the letters of the alphabet. There is no indication of what happens if higher numbers are required, the dimension might as well be infinite.

ways to divide the board, for example, we could take "black and white and the schema of squares" as simples out of which the whole is composed (§47). Likewise, monochrome patches would be considered "simple" in most games that deal with colours, but if we talk about flags, the French *Tricolor* and the *Union Jack* do not need to be analysed into their constituents, but are treated as units (cf. §64). What a simple object is, a possible location in grammatical space, depends on the purpose of categorisation.

The problem of relative simplicity is connected to the problem of objects and names: The starting point of these considerations is that what is ordinarily called a simple "thing" (referred to by a name) can be destroyed (e.g. the sword that is called "Nothung"). However, sentences using the name still have sense when the thing is destroyed, or even if it never existed: "Nothung has a sharp blade" (§39). Therefore, it seems, "normal names" cannot be real names; what the *real* name refers to, the *real* object, must be that which cannot be destroyed. Notoriously, Wittgenstein does not give a definite account of his position, but a reply along the spatial lines we have highlighted could be reconstructed like this: a word is a possibility in a space; the "object" it talks about is the special kind of existence for which destruction is not even possible, like a geometrical point. This would be not too far from the Tractarian account of objects ("Anyway, the thing about progress is that it always seems greater than it really is"). In this sense the argument is not directed against his former view. But it is in another sense, i.e. the following.

Like the objects of the *Tractatus*, the "things" discussed in the *Investigations* are supposed to be simple. However, they are no longer supposed to be absolutely simple as the simple "points" of an absolute logical space were. Instead they are only simple relative to a given space in which the standard of simplicity is such that their "constituents" cannot even be mentioned. In the "space of swords", Nothung can indeed be considered a simple, but its only purpose is to distinguish swords from other swords. In most cases, these spaces are, of course, more complex and have further dimensions so that we speak also of blade and handle of a sword, or of its colour and shape, and in this sense it is composite. However, simplicity and compositeness are always context relative:

> If I tell someone without any further explanation, "What I see before me now is composite", he will legitimately ask, "What do you mean by 'composite'? For there are all sorts of things it may mean!" – The question "Is what you see composite?" makes good sense if it is already established what kind of compositeness – that is, which particular use of this word – is in question.[154] (§47)

154. In this last sentence, the word "use" is quite close to my reading of "space". This shows once again the contrast to an account such as Engelmann's which highlights the difference between the spatial grammar of the middle period and grammar as use in the late period. Both have the function of providing a context and both are

The danger Wittgenstein saw in his former account – propositions being pictures of facts in logical space – was the reification of possibilities. Speaking of a "substance", which is indestructible and remains fixed while the configurations of the objects change, admittedly seems to imply that objects are, well, *substantial* or even *substantival*. The argument in the §§50s of the *Investigations* is basically a specification, an extension of the *Tractatus* idea that these objects are not to be confused with physical objects, but belong to the geometry of logical space. The difference is that now the idea of a universal logical space is rejected and that therefore what counts as a simple object depends on the respective space it is used in. In the space – the game – of measuring length, the prototype metre is (or used to be) part of a *geometry* with all its features we have elaborated so far: it constitutes the possibility of other things to be measured in metres, but it cannot be measured in metres itself; for it is inconceivable that it turns out that it is not one metre long. This would only be possible if the standard of a metre had changed. If the metre is defined as the length of a path travelled by light in a vacuum in a certain time, it is possible to say that the physical platinum bar in Paris is not exactly one metre long. But this possibility rests on the change of standard, in which case the prototype metre would have lost its role.[155]

In an analogous sense colours are "indestructible". Like the prototype metre is neither one metre long nor not one metre long, the colour "red" does neither exist nor not exist. However, this special status of being beyond ordinary existence, of being "timeless" or "indestructible" (§58) could better be expressed as red being part of a language-game which is, in fact, played. "Red exists" would then be another way to say that "red has meaning" or "red has a role in this game". This latter statement is a statement about the use of the word "red", not about the metaphysical nature of red. Hence, the argument is directed against the reification of grammar as a special sort of existence, which the space-metaphor is perhaps prone to, and suggests an understanding of meaning as moves in a language-game. This shall become more relevant when we speak about rule-following (in 6.1.3).

The plurality, that there is not just *one* way in which words or propositions function, is clearly a criticism of the undifferentiated position in the *Tractatus*. There were "forms of objects", such as space, time and colour (cf. TLP, 2.0251), but these were all unified in the

considered pluralistic concepts. Engelmann sees that use and grammar are similar, but assumes that grammar has completely changed its meaning and has nothing to do with the "old grammar" of the *Big Typescript*.

155. This accounts also for Kripke's argument that the bar can be measured in inches (for which he assumes a different standard) and if it is 39.37 inches long then why should it not be one metre long? (cf. Kripke 1980, 54). The point is that if the proposition that the bar is 39.37 inches long is an expression of a definition, of the translation rule from metres to inches, then the expression is no more informative than saying that the metre bar is 100 cm long. If, however, it is an empirical observation that the bar is 39.37 inches long and one wants to say that *therefore* it is 1 m long, then one takes the standard for what is 1 m long to be dependent on the inch standard and defines it as 39.37 inches. In this case the metre bar in Paris would again have lost its role as the standard of measuring in metres.

most general *logical form*. In the *Investigations*, Wittgenstein repeatedly rejects views which strive for a single and general form of words (cf. §1) and sentences:

> There are *countless* kinds; countless different kinds of use of all the things we call "signs", "words", "sentences". And this diversity is not something fixed, given once for all; but new types of language, new language-games, as we may say, come into existence, and others become obsolete and get forgotten. (We can get a *rough picture* of this from the changes in mathematics.)
>
> The word "language-*game*" is used here to emphasize the fact that the *speaking* of language is part of an activity, or of a form of life.
>
> [...] [long list of examples of different language-games]
>
> It is interesting to compare the diversity of the tools of language and of the ways they are used, the diversity of kinds of word and sentence, with what logicians have said about the structure of language. (This includes the author of the *Tractatus Logico-Philosophicus*.) (§23)

This plea for plurality is followed by the considerations of naming. The discussion of ostensive definitions centres around the question in which space a certain definition is made. After reaffirming that a proposition is always a possibility in contrast to other possibilities in the same space (cf. §20), the diversity of spaces turns out to be a problem. For the bare expression "this" or the pointing of a finger is empty if one does not know the space in which "this" is supposed to highlight one location among others. It is acknowledged, that "one can ostensively define a person's name, the name of a colour, the name of a material, a number-word, the name of a point of the compass, and so on" (§28) – but in many cases, even those enumerated here, the object in question would be defined by the *same* gesture. To take again an example from the middle period, let us consider the example of the unknown word "tove" in the *Blue Book*:

> Let us then explain the word "tove" by pointing to a pencil and saying" this is tove" [...] The definition then can be interpreted to mean: – " This is a pencil"," This is round"," This is wood"," This is one"," This is hard", etc.. etc. (BB, 2)

In the example in the *Investigations*, the envisaged space is the space of natural numbers and the place in it which shall be defined is the number "two" – by pointing to two nuts. The word "number" would help preventing misunderstandings insofar as it "shows what *place* in language, in grammar, we assign to the word" (PI, §29). The place of "two" is not a variety of nuts among others or their size or colour or whatever quality the nuts might have (we

see that there are *"countless*" language-games in this sense), but a place in number space and as such distinguished from other numbers: these nuts are "two" as opposed to "one", "three", "four" etc. Familiarity with number space, with the concept of number and counting, is a requirement to understand this kind of definition. Wittgenstein also asks wherein the knowledge of a space consists and anticipates ideas about the crucial role of the use of language – but even besides these complications, the argument is a good example for the presence of spatial thinking in his later philosophy. He repeatedly highlights the importance of the "preparation of a place" (e.g. §31) so that the ostensive definition can have sense:[156] the preparation of a place where the object belongs. And in the same remarks the terminology fluctuates between spatial metaphors and game-terminology: an ostensive definition can be understood if one knows the "place" of the word, or "what role it is supposed to play" (§30); one must "already master a language-game" (§33). Again chess pieces in chess space serve as an analogy (§31).

One central aspect of many of the following remarks is the attempt to undermine the generalising tendency of Wittgenstein's early philosophy. In passages that I have already mentioned in the previous, he points out: That the simplicity of objects is not stable, but depends on the contextual framework which is now crucially tied to linguistic practice and use (beyond the already mentioned remarks see also §§59–64). That the concept of "game" is a family of games whose members need not have one thing in common – it is intrinsically an open concept and cannot therefore be fixed as a disjunction of "all" games (§§66–67). That even the seemingly meta-logical word "rule" allows for many different interpretations which can likewise be exemplified with the loose and indeterminate rules of games which nevertheless and undoubtedly count as games; rules needn't be fixed but can also be made up, and even altered, "as we go along" (cf. §§82–84). That proper names can be understood in many different ways, and different (Russellian) descriptions will do depending on the context (cf. §87). That the meaning of "exactitude" may vary from one language-game to another (cf. §88). As does the meaning of "analysis" (cf. §§90–92). That hence there is no "final analysis" of language, no "*single* completely resolved form of every expression" (§91). That Wittgenstein's "grammatical" investigation is indeed concerned with the *possibilities* of phenomena (again §90), but not with the "a priori order of the world: that is the order of *possibilities*, which must be common to both world and thought" (§97).[157] That, unlike

156. A "preparation" in the same sense in which the definition of simultaneity was a preparation of measuring two events in special relativity. Or like the expectation was the preparation of the place in grammatical space for the expected event (cf. section 4.5).

157. I take it that the emphasis on *possibilities* qualifies the expression "a priori order of the world" and illuminates the relation between *Tractatus* and *Investigations* in this point: while there was "no order of things a priori" (TLP, 5.634) in the sense of substantial truths, there was still an *a priori* order of *possibilities*, which is implied by the expressions of logical form and logical space. Now the *Investigations* reject even this form of

Nicod's simple geometries, Wittgenstein's simple language-games are not "preparatory studies for a future regularization of language – as it were first approximations" –, but *objects of comparison*, used like a measuring rod, not as a "preconceived idea to which reality *must* correspond". That the particular order, the particular system or space or game, is established "with a particular end in view; one out of many possible orders; not *the* order" (§§130–132). That the "crystalline purity" of logic, which I take to suggest a system of static elements (crystalline molecules don't move) is not *the* only possible way to look at things, but *one* form of representation (§§107–108; cf. MS157b, 3v: "The a priori must become *one* form of representation").[158]

5.4 Larger Frameworks

5.4.1 Forms of Life

The word language-*game*, it is said in the *Investigations*, is used "in order to emphasize that the *speaking* of a language is part of an activity, or of a form of life" (§23). Hence forms of life obviously have a larger scope than language-games. To be sure, from the few remarks that occur in different manuscripts, it is not possible to extract a developed concept. (The other concepts can be very diverse too, but there is at least a larger textual basis in the manuscripts to see something like a prototypical use). Nevertheless, the usage of the concept in those remarks does fall into my range of interest with regards to spatial frameworks. I suggest seeing forms of life as descendants of earlier „forms": of the Kantian forms of intuition, Tractarian logical form, late Wittgenstein's forms of expression and representation. In this row, however, they take up a peculiar place.

One indication of them belonging to the group of concepts I am interested in, is that they are not to be questioned. They resemble the constitutive frameworks we have hitherto encountered by not being candidates for truth and falsity, but instead determining those candidates. After having discussed the irreducible nature of following rules, Wittgenstein calls the shared practices of rule-following, e.g. in mathematics, the "scaffolding from which

a unique order *a priori* – there are instead many orders of possibilities, but they are only *a priori* insofar as we are committed to accept them as frameworks: within the framework some things that belong to its geometry can then indeed be said to be independent of experience, but no metaphysical truth is expressed in this utterance. It is a description of forms of expression – and this is obviously not Kant's use of the term anymore, although there is a relation.

158. Etymological arguments are not generally Wittgenstein's style, but his emphasis on "*Vorurteil*" in §108 reveals a striking ambivalence in this context: the crystalline purity is *a priori* insofar as it is *prior* to any *judgment* ("*vor* dem *Urteil*"), but it also captures the negative connotations of the word "prejudice" insofar as it needn't be true.

our language operates"[159] (PI, §240). The conventionalist tone of this remark is immediately countered:

> So you are saying that human agreement decides what is true and what is false?" – What is true or false is what human beings *say*; and it is in their *language* that human beings agree. This is agreement not in opinions, but rather in form of life. (§241)

What is at the basis of our practices is not that people happen to agree on the truth of certain facts (agreement in opinions), but on the question over what *can* be true or false. This agreement is embodied in the way of acting and in the practices of human beings which Wittgenstein here calls forms of life. The appearance of empirical contingency, which this concept certainly has and of which more shall be said in the next chapter, is at least weakened by the word "form" which, in light of Wittgenstein's previous use of formal concepts, gives it a kind of *logical* connotation. The same reminder is in place when looking at another mention of forms of life that can also be explained with their constitutive function: "What has to be accepted – the given – are *forms of life*" (PPF, §345). Instead of trying to find an ultimate foundation of knowledge in sense-data (the "given" for phenomenologists), Wittgenstein points out that all that has to be accepted is the framework which first of all enables any sort of meaningful discourse or action. In this remark, the plural is important: otherwise it might suggest an unquestioning acceptance of the one framework or form of life one is placed in. However, the plural is a reminder of the possibility of different frameworks which may not yet exist, but which become conceivable once the current framework is sufficiently clarified. In order to change the status quo, one first needs to accept it. One can indeed imagine other forms of life which Wittgenstein explicitly mentions in combination with imagining languages: "to imagine a language means to imagine a form of life" (PI, §19).[160] Anyway, the remark is not made in the context of politics and tradition, but in the context of mathematics in which the term "given" also has its place.[161] Mathematics as a science depends indeed on the shared practice of mathematicians and, perhaps even stronger, on

159. Note that also in the *Tractatus* "spatial" concepts were described with the image of "scaffolding" (cf. TLP, 3.42; 4.023; 6.124). A scaffolding is something stable and necessary from which more dynamic and optional movements can be performed. Juliet Floyd also hints at the contrast to Hilbert's metaphor of "*Fachwerk*" which is integrated into the building, whereas scaffolding can be removed and rebuilt in new ways while the building stays the same (private conversation, cf. Floyd 2018, 78). Thus the logically conventionalist element of "spaces" would be accounted for with this imagery, too.

160. Wittgenstein mentions languages that consist only of orders or questions (§19). One could also think of something like Jorge Luis Borges' story *Tlön, Uqbar, Orbis Tertius* where a people is sketched that has a language tailored to radical philosophical idealism. Imagining these languages can also help us get clearer about our own form of life.

161. I owe this observation to Felix Mühlhölzer.

ordinary people who engage in mathematical activities. At the same time "real mathematics" in Wittgenstein's sense consists not in just acting in accordance with inherited rules, but in developing one's own rules. In doing so, one is at the same time acting within a form of life – the cultural practice of mathematics – as well as changing its rules.

5.4.2 Style and Picture

The notion of "style" can be understood in two senses both of which are present in Wittgenstein's writings as Sabine Plaud (2011) has pointed out. First, it is used in the sense of the style of an individual, the aesthetical appearance of an artefact which recognisably links it to a specific person. The relevance of style in this sense can be seen in Wittgenstein's emphasis on his style of writing which is reflected in the endless revisions of his remarks as well as in explicit comments on style. While these often bemoan his shortcomings against an apparently very high stylistic standard, some are more reflective and indicate something super-individual about style: "Style is the expression of a general human necessity" (MS183, 23). "To write the right style is to place the carriage exactly straight on the rails" (MS117, 225). It is striking that stylistic matters seem to be closely related to the notion of grammar as delineated in the preceding chapters. When discussing grammatical differences, Wittgenstein describes his attitude as a special sensitivity towards stylistic matters:

> In a certain sense for us – that is in grammar – there are no "small differences". Anyway, the word "difference" means something completely different here than when we speak of differences between things.
>
> A philosopher feels changes in the style of a derivation which a contemporary mathematician passes over calmly with a blank face. (TS213, 643r)[162]

In the *Investigations*, such a change of style – not in mathematics, but in philosophical language – is characterised as a "grammatical movement", as a new way to conceive of, in this case, sensations. In fact, Wittgenstein suggests that comparing this move to a style is preferable to the expression "discovering a grammatical movement", presumably because it sounds too realist (cf. section 6.2.2): "Above all, you have found a new conception. As if you had invented a new way of painting; or, again, a new metre, or a new kind of song" (PI §401).

The collective rather than individual account of style, which shines through these remarks, hints towards the second sense of "style" which Plaud characterises as "a framework or

162. Note the new context of this remark after its transferral from MS111 where this connection to styles was missing (quoted in section 5.2.1).

as an a priori pattern of thought" (2011, 77). She shows this second sense to be rooted in the German tradition, in art history and cultural history in general. Oswald Spengler, for example, understood style in this second sense which may well be one of the ways in which he influenced Wittgenstein – his name is on the list of influences with Hertz, Boltzmann, Russell, etc. (see chapter 2). My claim is that this sense is connected to Wittgenstein's usage of "spatial" concepts which, like "style", can be conceived as frameworks that *a priori* constitute possibilities of thought and expression. For Spengler, a style is connected to his notion of culture which according to his historiography in *The Decline of the West* rise and fall and follow each other according to certain repeating patterns. The style of a culture determines the way in which it expresses and organises religious, intellectual, social or economic matters; hence it is not something individual, but a collective framework (cf. Plaud 2011, 86–89).[163]

Spengler understands the style of a culture very similar to what he calls a culture's "world picture" (*Weltbild)* or also *Weltanschauung*. An interesting echo of this is found in Wittgenstein's mention of Spengler in the pre-stage of the *Investigations* when he describes the method of "perspicuous presentation" as central to his own style and considers: "Perhaps a kind of the 'Weltanschauung'. Spengler" (TS220, 80).[164]

In her article, Plaud elaborates her intuition that world picture and style belong together for both Spengler and Wittgenstein, and I agree: both concepts belong to the orbit of *a priori* frameworks, spaces etc. However, *picture* plays a very versatile role in this whole conceptual field. We have already seen that Hertz's concept of *Bild* belongs to it. Like Hertz suggested a new *Bild* in his *Principles*, Wittgenstein suggested a new *Bild* in his *Tractatus*, a picture of which he later said that it "held us captive" (PI, §115). As it stands, this use of *Bild* is a bit obscure. Is it a simile, an analogy? Or is it a framework of possibilities, a space? In the latter case, being held captive by it would amount to hypostatising a form of representation to an *a priori* necessity of reality. "One thinks that one is tracing nature over and over again, and one is merely tracing round the frame through which we look at it" (§114).

163. A strong influence on Wittgenstein can be seen in his pessimistic sketches from 1930 which have been published by his editors as the preface of the *Philosophical Remarks*. In these remarks, Wittgenstein follows Spengler's tenor and terminology by speaking of his time as "civilisation" and *Unkultur*, words which characterise their time as the late stage of a culture in decline (cf. MS109, 204–5).

164. In the final version, this takes the form of a question: "Is this a 'Weltanschauung'?" (§122). Wittgenstein is here not objectively considering or diagnosing faulty frameworks, but conceives of his own world picture as one out of several possible ones. This question is perhaps another reason not to dogmatically take Wittgenstein's own method as *the* way to do philosophy, but as a suggestion that proves fruitful and seems appropriate to him. Kuusela also rightly points out that this "method of perspicuous presentation" may be *one* method, but perspicuity is achievable in various ways and therefore there is no contradiction to the remark that there are in fact "methods, different therapies, as it were " (PI, §133; cf. Kuusela 2008, 269).

A *Weltbild*, a world picture, can hold us captive, too, but has a larger scope than analogies that determine the representation of rather specific situations. Instead, world picture seems to be connected to forms of life, frameworks that determine the way we look at the world as a whole. Indeed, Wittgenstein uses the word like this in *On Certainty* where it is almost interchangeable with style and form of life (also the "spatial" terminology of "scaffolding" reappears).

> The existence of the earth is rather part of the whole *picture* which forms the starting-point of belief for me. (OC, §209)

> Now it gives our way of looking at things, and our researches, their form. Perhaps it was once disputed. But perhaps, for unthinkable ages, it has belonged to the *scaffolding* of our thoughts. (Every human being has parents.) (OC, §211)

What is interesting about these large framework in the last writings is that Wittgenstein speaks of them as forming a system of undoubted propositions. Importantly they feature again some kind of systematicity or even totality, although it is clear that it is not a totality in the *Tractatus* sense. The frameworks again constitute what cannot be doubted within it and they again show the systematicity we have marked as a criterion of spatial frameworks in the wider sense:

> When we first begin to believe anything, what we believe is not a single proposition, it is a whole system of propositions. (Light dawns gradually over the whole.) (OC, §141)

> It is not single axioms that strike me as obvious, it is a system in which consequences and premises give one another mutual support. (OC, §142)

> Our Knowledge builds a great system. And only in this system has the particular the value which we attribute to it. (OC, §410)

These kinds of framework are again characterised as a kind of mythology which was already a typical expression for the frameworks of notations and symbolisms. Wittgenstein here alludes to the ambivalences of frameworks that I shall discuss in the next chapter. On the one hand, the frameworks are unquestioned certainties which are beyond true and false, but determine the candidates for truth. On the other hand, they are themselves "inherited" and learned "practically" which implies that they themselves could be different: only their *role* gives them the seemingly necessary, "geometrical" status.

> But I did not get my picture of the world by satisfying myself of its correctness: nor do I have it because I am satisfied of its correctness. No: it is the inherited background against which I distinguish between true and false.
>
> The propositions describing this world-picture[165] might be part of a kind of mythology. And their role is like that of rules of a game; and the game can be learned purely practically, without learning any explicit rules (OC, §§94–95).
>
> It is clear that our empirical propositions do not all have the same status, since one can lay down such a proposition and turn it from an empirical propositions into a norm of description.
>
> Think of chemical investigations. Lavoisier makes experiments with substances in his laboratory and now he concludes that this and that takes place when there is burning. He does not say that it might happen otherwise another time. He has got hold of a definite world-picture – not of course one that he invented: he learned it as a child. I say world-picture and not hypothesis, because it is the matter-of-course foundation for his research and as such also goes unmentioned. (OC, §67)

The last remark indicates that Wittgenstein also assumes some influence of world-pictures on the sciences. This opens up some interesting connections between Wittgensteinian thinking and his contemporaries, especially Ludwik Fleck's theory of styles of thinking. Via Fleck the idea that scientific practice is constituted by the framework of possibilities in which it is pursued influenced Thomas Kuhn's theory of paradigms which may have some more similarities with Wittgenstein's grammar than generally assumed. In the last chapter, I will also indicate ways in which the idea of constitutive frameworks is at work in the debate on the "relativized a priori" in science. This debate, too, originates in the abstraction of space and geometry and led to the assumption of local *a priori* principles in science (cf. Stump 2015).[166]

5.5 Summary

I have investigated a number of concepts in order to show that spatial thinking and, related to it, grammatical thinking is not given up completely after the middle period, but that its

165. The translation is not consistent: in both cases ("picture of the world", "world-picture") the German is *Weltbild*.

166. I give a more detailed account of this relation in my forthcoming paper *Beyond "logical absolutism". Wittgenstein and the relativized a priori*.

functions had been shared with a number of concepts before the *Big Typescript* some of which survive until the very last writings. The variety of usages for the space concept, which I discussed in chapter 4, is expressed here through the fact that all these related concepts are pluralist concepts. Wittgenstein did not strive for just one notation (the phenomenological language was really just a short project he was very sceptical about from the start), but his use of notation is compatible with the pluralist and creative account of space, form, theory etc. which I discussed. Likewise, he did not strive for a "comprehensive grammar" which would account for all kinds of language use; grammar has always been differentiated which shows itself in the pluralist concepts that are related to it.

The investigation of these concepts has led to two problems which have implicitly already been mentioned: first, the problem of the static nature of space; second, the problem that logic is supposed to be independent of all experience, but that sometimes empirical propositions function like logical ones. And that the only justification for their logical status is: their empirically observable use in language. These two problems shall be discussed in the next chapter.

Chapter 6

Limitations of Spatial Frameworks

Nicht unfruchtbar wäre es, die Geschichte der neueren Philosophie unter dem Aspekt zu behandeln, wie sie mit dem Antagonismus von Statik und Dynamik im System sich abfand

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Negative Dialektik

The disintegration into several concepts reflects an increasing dissatisfaction with the conception of language as organised in space-like *a priori* structures. I have argued, however, that instead of simply being dismissed, many of the functions of spaces are taken up by other concepts as has been shown in the last chapter. In the following, I shall look closer at this transformation from spatial frameworks into more dynamic and flexible frameworks, but also how they lose their strictly *a priori* status and become intertwined with experience.[167]

I shall first look at the role of space's sister concept time which becomes increasingly important in the 1930s. The word-frequencies and contexts discussed in section 5.3.1 suggest that calculi and games *replace* spaces and this raises the question why they do so and how they are preferable. As has been shown, they are not something completely new, but share important features with spaces. However, they differ in being more dynamic than the static spaces of possible locations. After briefly sketching the literal occurrences of time in Wittgenstein's writings – mainly investigations of time in terms of spatial concepts – I shall reverse these priorities in the second section and look at the transformations of spatial

167. *A priori* is here understood in a minimal sense as independent of experience – without the Kantian apodicticity.

frameworks under a temporal perspective. The contrast between a space of static possibilities and dynamic games shall be central.

In the second part of this chapter, I look at another consequence of this temporalisation. By becoming temporal, spaces have also become historical. By having become changeable, constitutive frameworks now appear as historical facts which can themselves be studied like empirical objects and events. The important tension between their function as logical, *a priori*, and constitutive frames on the one hand and their own empirical nature and (implied) fallibility on the other hand shall be studied by means of two contrasting juxtapositions: the first distinction is one between a prescriptive understanding of rules and a descriptive approach to regularities. The second is one between grammatical realism and an emphasis on creativity and construction. My view highlights the ambiguities in each of these distinctions: Wittgenstein refuses to take a clear position to either of them and acknowledges the descriptive as well as prescriptive and creative aspects of language.

6.1 Space and Time

In light of the transition from spaces to calculi and finally to games, it seems that time plays a crucial role in Wittgenstein's later work. In this section I try to answer the question what that role might be, albeit in a different way than one might expect. For the short answer is that time is not as important as space as a methodical tool for Wittgenstein. Although it does repeatedly occur as a problem in his writings, I cannot see the systematic unity in its treatment that I suggest with regards to space. Before investigating how it does nevertheless affect my reading of spaces as grammatical frameworks, I shall look at some examples in which time and temporal issues are dealt with. In this excursus, I highlight the remarkable absence of time in the *Tractatus* as well as some arguments about time which seem to be relatively isolated in Wittgenstein's oeuvre.

While these issues deal more or less directly with time, they touch my actual research question only in an indirect way. After the excursus, I shall turn the focus back on space and spatial frameworks, but under the aspect of their being implicitly affected by considerations of time. My claim is that the disappearance of space as a central concept after the middle period is motivated by the increasing acceptance of language as a temporal phenomenon. The first step to substantiate this claim is to see what happens to the concept of spatial frameworks as the terminology changes: I suggest that the emergence of new concepts, notably calculus and language-game, is related to the disappearance of space and that they are, at least partly, introduced in order to replace it. As we have seen, they share central features with spaces, but differ by being essentially temporal frameworks, insofar as they treat of moves instead of

static locations. Moreover, the frameworks themselves are considered changeable. Thus the "objects" in space as well as the "geometry" itself become dynamic.

Calculi and games are rule-governed frameworks. The problem of rule-following can be conceived in terms of the clash of the two images we have encountered: on the one hand the static space-concept which assumes all of its possibilities to be simultaneously, or rather timelessly, determined by geometrical rules; on the other hand the concept of dynamic games in which we can never quite anticipate the *application* of the rule, at least not *all* applications, as they depend on unforeseeable interpretations for future situations. As language is increasingly conceived in terms of use, the static picture of space becomes less attractive. The problems of generality and intentionality shall be readdressed in this section as well as the shifting use of the term "rule".

6.1.1 Excursus: Wittgenstein on Time

It is striking that of Kant's two forms of intuition only space had this fertile career in mathematics, philosophy, and the sciences, whereas time has often been treated as somewhat mysterious or simply subsumed under spatial concepts. This is in fact what happened to time in the *Tractatus*.[168] Scholars have noticed that its system, and notably the notion of logical space, is essentially a *static* affair, especially if considered in contrast to Wittgenstein's later philosophy (cf. Hyder 2002, 196; Pilch 2017, 23; Sluga 2017, 417). On the few occasions where time is mentioned, it is treated in a spatialised way, as just one dimension of logical space which itself is timeless, a "form of objects" (cf. TLP, 2.0121; 2.0251) where "form" is closely related to the idea of a space of simultaneous possibilities as we have seen in chapter 3.[169] Later in the *Tractatus* there is a brief hint to a vaguely Kantian understanding of time as form which cannot itself be experienced and therefore cannot be compared to processes *in* time (TLP, 6.3611). The distinction between eternity and temporal infinity as well as the relegation of ethics and value to an unspatial and untemporal (and ineffable) realm seem to be motivated by a Kantian reasoning, too (TLP, 6.431 ff.).

168. It is possible to highlight certain remarks in the *Tractatus* that stress the activity character of thinking (TLP, 3.326; 3.5) and the operational account of logic. This would make language already dynamic in the early philosophy. While there may indeed be such an intuition already contained in the *Tractatus*, I do not think that Wittgenstein was aware of it because the overall impression, not least the space imagery, is decidedly static. This static imagery may of course be one of the "steps of the ladder" which are, according to some resolute readings, to be understood as transitory nonsense. But given Wittgenstein's later use of spaces and his self-criticism, I do not consider this a plausible reading. In any case, the temptation to conceive of possible applications as predetermined and reified locations remains important for Wittgenstein in his later writings.

169. This tendency to conceive of time in spatial terms survives until Wittgenstein's very last writings: "A description is a representation of a distribution in a space (in that of time, for instance)" (PPF, §70).

In Wittgenstein's middle period, problems with time occur in various contexts. One rather puzzling strand of arguments comes up in September 1930. In these passages, a series of oppositions is presented which are, as so often in Wittgenstein, understandable enough in themselves, but it is not clear *why* these problems are raised in the first place. The main contrast is between time and truth-functions and the criterion for their comparison is their respective universality or generality: can both be applied to all kinds of proposition? Wittgenstein expresses an intuition that they cannot, that time is, in some sense, less general than negation or disjunction.[170] What comes closest to a proof of this intuition is that mathematical propositions and some immediate – to use a slightly inapt expression – protocol sentences ("The sky is blue.")[171] are timeless, but the truth-functions can still be applied to them; hence time would be less general than truth-functionality. But more often the difference is presented as a kind of feeling: time "tastes" of content, truth-functions of form; the former is "coloured", the latter "dim";[172] time corresponds to what is pictured, truth-functions to the frame of the picture; time is phenomenological, truth-functions are logical (cf. MS109, 120–133). These remarks are transferred to TSS211 and 212 and finally form a sub-chapter of the *Big Typescript* with the title "Is time essential to propositions? Comparison between: time and truth-functions" (TS213, 113–116). This seems to give them some relevance, as

170. These are apparently treated as truth-functions here, not as operations – thereby deviating from the terminology of the *Tractatus*. Then again, this may be explained with Wittgenstein's focus on the difference between what he calls the "logic of content" and the "logic of form" (MS109, 130) instead of the difference between functions and operations which both belong to the logic of form (It remains unclear what the "logic of content" is and this may be the reason why Wittgenstein does not come back to this issue after 1932).

171. Obviously, the use of these sentences is less scientific than the Vienna Circle's protocol sentences which were meant to record a given observation *at a certain time* What they have in common is their supposed certainty as opposed to constructed propositions according to hypothetical rules. Carnap developed his ideas of a protocol language at about the same time. He published them in his essay *Die physikalische Sprache als Universalsprache der Wissenschaft'* (1931) in which Wittgenstein recognised his own "work in progress" which he had shared with members of the Vienna Circle. He particularly insisted that the notion of hypotheses, which, unlike protocol sentences, can never be verified completely, had been used by Carnap without referencing its origin. As a consequence, Wittgenstein broke off his connection to Carnap and with the Vienna Circle in general, only occasionally meeting with Schlick and Waismann in the years to come (cf. Stadler 1997, 475–480).

172. Mihai Ometiţă, who offers a good description of the disintegration of logical space and the role of colours, misunderstands this distinction when he translates *bunt* and *matt* as "multicoloured" and "faint": he interprets faint as (intellectually, philosophically) weak and holds that this shows Wittgenstein's crumbling account of logical form which gives way to a "multicoloured – that is heterogeneous – logic" (Ometiţă 2017, 152). However, "bunt" is not used as opposed to monochrome here, but to German "matt" in the sense of English "dim" or "opaque"; far from being philosophically weak the temptation to conceive of logical form as a sort of dim shadow of reality remains strong in Wittgenstein in these years (in fact, he never abandons it completely despite recognising the problems of this imagery). See for instance this remark which also shows the connection to geometry: "The so-called geometrical line stands to a colour boundary not like something fine to something coarse, but like possibility to reality (think of the understanding of possibility as the shadow of reality)" (MS114, 15). Ometiţă is right in stressing the general opposition between universal logical form and local grammatical spaces; but in this case he is right for the wrong reasons.

the chapter headings provide a good overview over the topics that occupied the "middle Wittgenstein".[173] But one must not forget that this typescript was mainly a collection of material for further revision. The remarks about time and truth-functions undergo a first stage of revisions with handwritten addenda, but are not used any further in later projects, in the corpus of the *Philosophical Grammar*, MS116 or the various versions of the *Investigations*. As a consequence they are ignored even by scholars who work on "Wittgenstein and Time" (e.g. Hintikka 1996; Schulte 2006; Sluga 2017) – and this is understandable: those remarks may be worth a detailed study as a local topic of the "middle Wittgenstein", but it seems to be not much more than that.

Apart from these arguments, which mention time explicitly as an object of interest, temporal issues seem to have a largely illustrative purpose. Most occurrences are variations of the general method to distinguish different grammatical spaces. Scholars have, for example, noticed the contrast between physical time ("information time") and experienced time ("memory time") which correspond to different ways of verification (cf. Schulte 2006; Hintikka 1996). Different grammars constitute different conceptions of time: on the one hand a measurable sequence of distinct states and on the other an indeterminate continuum of past events leading to an ever moving end-point which is ontologically privileged and called "present". Like exactitude had different meanings in visual space and physical space, so time is understood differently whether it is verified by introspection or physical measurement. This was what should be exemplified by the analogy of the film-projector: the film-strip and the screen are different grammatical systems with different possibilities, possibilities of verification and possibilities of alternatives.[174] Reflections on time that are based on this train of thought also occur in the form of an argument on the seemingly fleeting reality which cannot be captured in language (cf. VoW, 321) or in a reply to Russell's idea that the world could have been created five minutes ago.[175] In both cases, Wittgenstein would point out the impossibility to account for the absolute exactitude that seems to be required for fixing a fleeting experience, or the absolute certainty that underlies Russell's speculations:

173. As a synopsis of the years 1929–1932, the *Big Typescript*'s structure represents the "late middle Wittgenstein", that is, the sections and sub-sections are titled according to topics which Wittgenstein considered important in 1932. Therefore the headings can help to understand what problems shall be addressed with the sometimes rather puzzling remarks. They are often literally identical or very similar to the manuscripts, but less understandable than those because they appear out of their original context; however, the section in the *Big Typescript* gives them a new context, not necessarily one of structured argument, but one of thematic belonging. The order of the remarks in each section may not always form a coherent argument, but the order of the *sections* is by no means arbitrary.

174. Jaakko Hintikka interprets this within a larger contrast between perspectival and ordinary identification, memory and experience belonging to the former, physical time belonging to the latter (cf. Hintikka 1996).

175. Contrary to Sluga's view, these questions are not only dealt with in Wittgenstein's last years (cf. Sluga 2017, 437), but already in his lectures from 1933 which are clearly related to his ideas about verification, sense, and grammar which he had developed in the preceding years (cf. MN 2016, 294).

in the absence of a method of verification it has no sense to strive for such an unreachable linguistic ideal. This warning against misleading ideals is a recurring theme in the writings from the early 30s; at this time Wittgenstein also starts quoting St. Augustine who shall become his most frequent reference to the philosophical tradition. A quote by Augustine from 1930 (cf. MS108, 272) is reused in the *Investigations* where the interlocutor wonders how a proposition can serve its "extraordinary" role in representing reality; to the typically Wittgensteinian answer "nothing is concealed", he would reply: "yes, but it all goes by so quickly, and I should like to see it, as it were, more fully laid out." (PI, §435). – And indeed Wittgenstein warns at this point about the temptation of a phenomenological language which is a philosophical "dead end" (cf. also MS113, 123v) and quotes Augustine's dictum that the words of time measurement are used frequently and without problems in ordinary language and yet remain obscure and mysterious as if they had to be discovered anew (cf. PI, §436; *Confessions*, book 11, ch. 22).

Book 11 of the *Confessions* differs from the preceding books by being less "confessional", less autobiographical, and more philosophical. Its topic is time. Wittgenstein takes another example from this chapter in order to illustrate his method. He quotes Augustine's question "what is time?", which is tied to the use of time expressions and methods of time measurement, first in 1931 (MS111, 137) and later in the *Investigations* where it is cited as an example of a "grammatical investigation" (§90).[176]

Similarly, Wittgenstein invokes Einstein as an ally in focusing on methods of measurement:[177] "what Einstein has taught the world: our method of time measurement belongs to the grammar of our time expressions" (MS119, 116v). And like Einstein, Wittgenstein aims to investigate "the clocks with which we measure the phenomena" (MS164, 82). In all these cases, time seems to be no more than an example for the requirement to look at grammar instead of searching for the essence of a concept, to think about ways of verification instead of a seemingly straightforward quest for truth; comparisons to metrics occur in all kinds of contexts of his philosophy and are not restricted to time measurement. Thus I shall in the following focus more on structural ramifications of the general move from abstract and static logical spaces to concrete and dynamic language-games (which should not be understood as an alternative *to* logic, but as a *new form* of logic: as something temporal and changeable). Therefore, I shall once again turn to calculi and games as the dynamic descendants of static spaces.

176. It is perhaps significant that Augustine's puzzle is based on a confusion of measurement of time with measurement of space. However, one must be careful not to interpret too much into Wittgenstein quotes of other thinkers. They often take a very specific role in the new context which can be quite remote from the original one.

177. For a detailed study of Wittgenstein's references to Einsteinian time metrics see Kusch 2011.

6.1.2 Dynamic Frameworks: Calculi and Games Reconsidered

We have seen in the last chapter that "calculus" accounts for a number of the functions of "space". What are the differences? Why does it seem to *replace* "spaces" as the frequencies of their occurrences suggest? Wittgenstein never makes them explicit, but there are some, as it were, analytic differences, in the concepts themselves. One is that, unlike to "space", there is a verb to the noun "calculus", namely "to calculate", *kalkulieren*.[178] So unlike space a calculus is inherently connected to something that is *done*, not something that timelessly stays in its place. Like spaces, calculi account for possibilities, but these possibilities are possible "steps" or "moves", in any case *activities* which are actually performed, not abstract logical locations in a formalised space. To be sure, a calculus is still a very formal concept and one can, in principle, abstract from concrete calculations; but the mere possibility of conceiving in terms of activities what formerly was conceived in terms of static locations moves the idea of grammar away from "space" and towards "language-games" in which we have to do with activities without any doubt.

> The notion of the proposition, hence of understanding and of the thought, must certainly justify the possibility of the calculus. And the calculus takes place in time, it is – so to speak – spread out. (MS109, 182)

> Is it like this: Only language as a phenomenon that is extended in time has grammar. (MS109, 288)

> Mathematics is a calculus and the calculus does not say of any sign that it was merely *possible*, but it deals only with those signs with which it *actually* operates. (MS113, 98v)

The last remark takes up the *Tractatus* idea that nothing in logic, and this applies to language as well as to mathematics, can be "merely possible". However, it does not connect this notion to a "modal realist" sphere of "all possibilities" (TLP, 2.0121), but ties this idea of actuality to actual use, to the *activity* of calculation.

What makes calculi more dynamic than spaces is that, while both are frameworks of possibilities, spaces constitute possible locations, whereas calculi constitute possible moves; the former refer to knowledge of a grammatical place, the latter to a form of "know-how", a capability to perform certain moves. This slight shift of emphasis brings up new problems

178. The connection of verbs and time shows itself in the fact, that the German word for verb is *Zeitwort*, a term Wittgenstein uses a lot in order to accentuate the – sometimes misleading – temporal character of verbs, e.g.: "The *Zeitwort* "to want" makes us compare the activity of wanting with the activity of carrying out what is wanted" (MS115, 106).

which shall become more relevant in the context of rules: *when* do I know how to do something? What does the "know-how" consist in? What is the grammar of "can"? And how are the rules of the calculus to be interpreted in individual cases?[179] In principle, these questions could be asked about knowledge of a place as well, but they are far less obvious. Knowing a place is much more conceived as a sort of permanent state of knowledge, while knowing how to *do* something raises a lot of questions as to what it is to understand a rule and to be able to follow it, which repeatedly come up in Wittgenstein's later thinking.

> What is at the basis of the word "can" in "I can now go on writing myself" is only the idea of the variable expression (hence, the idea of a *sign*, again only a tile in the calculus which itself only unfolds in time) and, say, the computations of a few further numbers "in the head". (MS110, 295)

Similarly, the rules of games are inherently connected to their applications, that is, to activities. In the late 30s, the connection from language-games to human activity is expressed in a number of remarks:

> The basic form of a game must be one in which actions take place. (MS119, 77v)

> To describe a language-game is to describe the *actions* of human beings [...]

> There is something in our language that we call the description of activities, things etc. *and also* something that we call the description of images, *impressions*. The 'description of a language-game' is of the first kind. (MS119, 147r–148r)

While the last remark mentions the ambiguity of "description" of which more shall be said in section 6.2, here the emphasis on *activity* is more relevant; it returns in the later statement that the word "language-*game"* indicates "that the *speaking* of language is part of an activity" (PI, §23). That the language-games with which Wittgenstein describes philosophical problems are dynamic is emphasised in other parts of the *Investigations* as well. We have seen that parts of the discussions in the first 30 paragraphs can be conceived in terms of constitutive frameworks whose internal oppositions are important criteria for their sense: the contextual space in which one thing is marked as opposed to others is a requirement for the thing (proposition, sign) to have sense. Different spaces with different oppositions are listed in §33

179. One way to put it is that up to this point "possibility" was the dominating problem, in the three senses of possibility mentioned in chapter 3, but also in the sense of grammatical spaces in the middle period which allow for different possibilities. In the course of the middle period, the interpretation of rules which seem to determine the possible applications straightforwardly, becomes more problematic: "Everything is now contained in the 'interpretation'. How the problem changes its house!" (MS109, 67).

(all involve the colour blue: which shows that the meaning is not contained in the sign "blue", but in the circumstances that open up the grammatical space of a situation). Here we find the first indication of a central problem in the relations of space, time, and grammar. One is tempted to conceive of the meaning of a word or a sentence as an entity or event which accompanies the utterance. We have seen Wittgenstein's initial inclination to understand meaning as something that depends on a system, as a place in a space, i.e. a place as opposed to other places in this space. As soon as the places are no longer simultaneously existing points, but temporally distinguished moves, these two conceptions – the timeless space and the temporal game – become incompatible. "Directing one's attention to something" does not only consist in a mental event parallel to some physical activity such as turning one's head; "these and similar things happen *while* one 'directs one's attention to this or that"'. But it also includes the circumstances of the utterance, what has happened before it and how the respective addressee reacts to it – and also how the words were used in the past and how people have reacted to it. These circumstances sum up to a pattern of normal usage which constitutes the meaning of the phrase in a given situation, but not to a space whose points are predetermined by a timeless geometry.[180] What does a move in chess consist in? It "doesn't consist only in pushing a piece from here to there on the board – nor yet in the thoughts and feelings that accompany the move: but in the circumstances that we call 'playing a game of chess', 'solving a chess problem', and the like."

While one may well call such an assembly of circumstances a "space", one decisive weakness of the space metaphor is its static nature. The notion of calculus already shifts the emphasis from possible locations to possible activities, and this shift is continued in the development of the game-metaphor. It further improves the idea of calculi by being more open and flexible while still keeping the notion of context-relative exactitude and necessity. The game metaphor is similar to space in that it illustrates the possibilities and the variety of different forms of frameworks (it illustrates the openness perhaps even better, cf. PI §§66–67); but it is more adequate with regards to practical questions insofar as the notion of rules is more relaxed and does not include a fixed entity "geometry" which – once in place – determines all steps rigorously and timelessly: "time" gets more important as the focus shifts towards the *application* of language in the later philosophy.

180. There is a sense in which grammar is timeless: Hacker insists that mathematical propositions and grammatical rules like "red is darker than pink" are not subject to temporal change (Baker and Hacker 2009, 279). Kuusela, with his focus on method, suggests that using a language-game as an object of comparison is to "freeze" a situation so that it is perspicuous enough to be described. This illustrates his point of simple language-games as tools for clarification, but it does not account for the inherently dynamic character of the concept of "game" nor for the change of language-game in the more complicated sense: the use of a word in all its variety (cf. Kuusela 2019b, 159).

6.1.3 Rules and their Application in Time

As shown, an important sense in which the analogy between language and games is illuminating stems from the fundamental difference between calculi and games on the one hand and static spaces on the other. Since the former treat of moves and actions, since language-games are linked to activities (cf. PI, §§7, 23), the question arises what knowledge of grammar, of the rules of a language-game, consists in.

The first assumption that leads to a static understanding of language is the idea of a proposition as a picture.[181] The second is the complementary metaphor of logical space which treats of a proposition, its negation, its implications, and possible combinations as locations none of which is preferred in a temporal or logical sense; all are equally present as possibilities. What the space metaphor suggests is hence a sort of simultaneous presence of the whole system. With the shift of emphasis from static pictures, which timelessly "exist" as possibilities in logical space, to systems of possible operations, moves, and actions the question of what it is to know the sense of a proposition becomes one of what it is to know how to *use* a proposition, to know which moves may be performed in a certain game. "The grammar of 'to know' is evidently closely related to the grammar of 'can', 'is able to'" (PI, §150a). Now, with the spatial metaphor in mind, it seems as if "to be able to" must consist in the presence of the whole system in one's mind. Admittedly, this is already hard to imagine within the space-paradigm since logical space was conceived as infinite, as is indeed Euclidean geometry; but the temptation is understandable with regards to limited spaces such as the logical space for two elementary propositions as used in the *Tractatus* (see section 3.2). These were supposed to be *complete* and each proposition, each location in it, was internally connected to all others. Likewise, in the middle period, the emphasis on the systematic character of conceptual frameworks seemed to imply, as Russell observed with regards to colour space, that making a colour statement includes something about "all the other colours" (Russell 1968, 168). As if a grammatical representation of colour space, for example the "colour-octahedron" (MS107, 282), was present as a whole in the mind of someone who speaks about colours.

Even though there were good reasons to introduce the metaphors of picture and space and to emphasise the systematicity of language in general, there are many ways to challenge this conception. One can, for example, point out the danger of reifying possibilities: if I *could* have said something, this *possibility* does not imply an *actual* presence in the mind – the method of the *Tractatus* was to treat all possibilities as if they were actual, but it did not speak of logic being "in the mind". However, this kind of psychologism appears as a

181. Consider for example Lessing's essay *Laocoon* (1873) where he famously divides the arts into static visual arts and the temporal arts of language which leads to different possibilities and limits.

recurring enemy or temptation in Wittgenstein's writings. The problem is addressed in the treatment of infinity and generality (are all, potentially infinite, possibilities that fall under a general statement real?) and exactitude (how can the inexactitude of the visual field be present in one's mind?). While these lines are discussed in some detail in the manuscripts of the middle period, the late Wittgenstein conceives of this problem in terms of temporal questions.

The game of chess has repeatedly been compared to a space or a calculus. There is, however, another, a different, angle from which Wittgenstein approaches this analogy:

> If I say, that knowing how to play chess consists in knowing the rules, is this knowledge of the rules in some way contained in every move? In a certain sense, it seems, yes! For otherwise a future experience was required to find out whether he is really playing chess; that is, "he is playing chess" would then be a hypothesis which could as such only be confirmed by experience, but never *proven*. On the other hand it seems that there is, in a certain sense, no doubt possible that I am playing chess and in this sense it has to be contained in what is now going on in my move. (MS110, 53)

"When *can* you play chess?" (MS110, 235) – this question is repeated many times throughout the *Nachlass* and also in the *Investigations*.[182] At one point, it is even compared to Augustine's attempt to understand time by means of the question "*when* do I measure a period of time?" (MS111, 137), but it is clear that we are speaking neither about chess nor about time itself, but about the temporal nature of language in a fundamental sense. While Augustine speaks *about* time, the question now is whether, in general, "meaning" or "knowledge" are themselves to be understood temporally. Is the knowledge of the grammar of a word a timeless *state*? A momentary *state of mind*? And how does this go together with the conception of meaning as *use* which is clearly not reducible to a single state? Do you know how to play chess "all the time? or just while making a move? And the *whole* of chess during each move? – How queer that knowing how to play chess should take such a short time, and a game so much longer!" (slip of paper, after §149).

The complete remark about the connection between knowing and being-able-to goes as follows:

> The grammar of the word 'to know' is evidently closely related to the grammar of the word 'can', 'is able to'. But also closely related to that of the word 'to understand' (to 'master' a technique). (§150)

182. The question occurs on a slip of paper which is clipped to the typescript of the *Investigations* TS227 after §149.

The presentation of the problem is indeed based on the concept of "understanding" as this word seems to mark the moment in which someone who is taught a game (a technique) grasps the rules "all at once". The remark on the slip of paper between §§149 and 150 contains more examples where one may wonder what the temporal status of knowledge and understanding is, particularly in comparison to what is rightly called "mental state" according to Wittgenstein: dejection, excitement, pain:

> "He was dejected the whole day"
>
> "He was in great excitement the whole day"
>
> "He has been in pain uninterruptedly since yesterday". –
>
> We also say, "Since yesterday I understood the word". 'Uninterruptedly', though? – To be sure one can speak of an interruption of understanding. But in what cases? Compare: "When did your pains get less?" and "When did you stop understanding that word?"

The grammar of "to know" is related to the grammar of "to be able to" and "to understand" and in all three cases the problems of time threaten the straightforward interpretation in terms of a space whose locations one could all "know", for example, by having a picture in one's head. "To understand" such a system would then seem to be an instantaneous grasping of the whole grammar, "as if one could, as it were, swallow it all at once" (MS114, 44v).

Baker and Hacker, with whom I agree that the rule-following problems are based on this picture of the static presence of a system (2009, 25), refer in their commentary mainly to MS119 where rule-following is discussed in a way very similar to the *Investigations*; but the underlying conflict is already present much earlier which is why I link it to the space and calculus idea. Baker and Hacker admit that the complex of problems is already present in the *Big Typescript* (2009, 8); this is, in turn, based on even earlier manuscripts, so the problem is in fact present in most of the "middle Wittgenstein". Even though they rightly point out that the issue got enriched in the *Investigations*, my emphasis is different. Since I am interested in the continuity of Wittgenstein's thinking and in the development of space concepts, I focus on the earlier sketches of the problem. The fact that the basic conflict of the rule-following considerations is already present in 1930 motivates my approach to relate it to Wittgenstein's way of thinking at the time which is indeed dominated by spatial metaphors.

With these considerations in mind, it makes sense to look at the *Big Typescript* again, the summary of the "middle period". The section headings can again be a helpful guide through the argument. The chapter "Instantaneous understanding and the application of a word in time" is divided into five sub-chapters:

1. To *understand* a word = *To be able* to use it. To *understand a language*: *to have command* of a calculus.

2. How does understanding a sentence accompany uttering or hearing it?

3. Is the meaning of a word shown in time? Like the actual degree of freedom in a mechanism? Is the meaning of a word only revealed in the course of time as its use develops?

4. Does a knowledge of grammatical rules accompany the expression of a sentence when we understand it – its words?

5. The rules of grammar – and the meaning of a word. Is meaning, when we understand it, grasped "all at once"? And unfolded (laid out, spread out), as it were, in the rules of grammar? (cf. MS213, 143–169; BT 2005, 110e-127e)

The tone of these headings is rather dubitative, no ultimate answers to the questions are given in the sections. In the last one, it even seems as if the two conceptions might coexist together so that meaning can be somehow compressed in the state of understanding and then unfolded in the rules. The tempting conception is illustrated with powerful images that suggest the static picture, the idea that the sign summed up the whole grammar as if it was contained in the sign "like a string of pearls in a box and that we only had to pull it out. (But this is precisely the image that leads us astray)" (MS112, 110v).[183] While this remark is set in the context of ostensive definitions of colour which seem to imply the whole grammar of colours, the same analogy is repeated in the context of negation in 1932 where Wittgenstein again points out that the sign of negation only has sense as a move in a game: "Only dynamically the sign functions, not statically" he says, followed by the string-of-pearls analogy, and he further comments:

> As if the understanding was a momentary grasping of something of which I can later draw the consequences; in a way, as if these consequences already existed, in an ideal sense, before they are drawn. As if the cube already contained the geometry of the cube and I only had to unfold it[184] [...] (MS114, 47v—48r)

183. Jane Heal sees the struggle against this image as central in Wittgenstein's work: "the idea of concept possession as the embodying of some psychic mechanism, in virtue of which a person is hooked up [...] to some predefined set of possibilities is exactly the tempting misconception from which the Wittgensteinian reflections are designed to free us" (1989, 207).

184. Note how something like this idea was implied by the remarks about possibility and space in TLP 2.012–2.013.

The problem can perhaps be summed up with this remark: "The most difficult problem seems to be the contrast, the relation, between the operating with language in time and the momentary grasping of a sentence" (MS111, 179). That Wittgenstein's reflections on rule-following are to a large extent motivated by these two contrasting conceptions is not a new observation; as mentioned, Baker and Hacker already point out that the indisputable fact that we sometimes do grasp something at a stroke, and call it understanding, clashes with the idea of meaning as use.[185] Beyond that, however, I want to point out that it is a conception Wittgenstein was prone to due to his earlier conceptions of meaning as locations in a conceptual space.

In the *Tractatus*, logical space was not dynamic. Time was simply a sub-form of logical form, one dimension of logical space, in which all possible facts coexist simultaneously, or rather timelessly. It is a variant of the block universe, in which every event has its fixed place, or rather a multidimensional multiverse in which all possible worlds (totalities of facts) are predetermined. Perhaps one should not reify this structure too much, but it is in any case obvious that time plays a very subordinated role in it and that the general conception is timeless.[186] The smaller grammatical spaces of the middle period in principle inherit this static nature, although the concept receives some first cracks. Especially the considerations of generality and intentionality seem to involve temporal questions: the problem was that an expectation is internally related to an event that has not yet happened (and might not happen at all). Again, it has to be stressed that the internal relation is one between possibilities, not actualities which could only be external. The similarity between a counterfactual proposition and a *capability* that is not put to action becomes pressing in this context.

A clearly timeless geometry was, for example, the space of colors which could be illustrated by the colour-octahedron, significantly a picture in which all colours can be perceived simultaneously as well as their relations to one another. The grammar of visual space, however, despite being frequently called a "geometry", did account for certain temporal phenomena which were related with their inherent inexactitude. Since the visual field is ever changing, its grammar does not allow for an absolute exactitude as could perhaps be achieved in a static, finite, and discrete geometry.

The problem becomes manifest in the context of understanding. Does he who understands a colour statement have the whole system of colours present in his mind? One could think

185. By taking into account large parts of Wittgenstein's *Nachlass*, I am closer to the exegetical approaches to rule-following like Baker and Hacker's than to its interpretation as an argument on scepticism as presented by Kripke (1982). Still, I go beyond them by showing where the idea of "understanding at a stroke" comes from. The philosophical confusions that Wittgenstein wants to clarify in his writings are above all views he is himself inclined to have, but fails to reconcile with other commonsensical views he accepts.

186. An attempt to understand time-relations in the *Tractatus* is given by Sluga, but he agrees that the book itself largely neglects temporal questions (2017, 427–429).

so because it is indeed a relatively perspicuous system and could exist as a schema on a piece of paper (a method Wittgenstein often uses: mental maps can be replaced with physical maps). But is having such a map the same as understanding colour concepts? Does he who understands a proposition have all its combinatorial possibilities present in his mind, the whole logical space through which it "reaches" (cf. TLP, 3.42)? Although a certain uneasiness with the idea of grasping something at a stroke can be seen already in remarks from 1930, Wittgenstein would at least consider that position which is probably based on his reflections on the systematicity of language discussed above: "If 'to understand' does not mean 'to translate', then it means: to see the sign in the space of its grammatical rules" (MS110, 51). Having said that each sign requires a grammatical space, Wittgenstein continues: "Hence the sign cannot exist without grammar" and further: "The sign without grammar would be the 'static' sign" (MS110, 126). I take it that at this time, which is, as it were, the beginning of the calculus phase, it dawned on him that the two images clash: that on the hand grammar is supposed to be a space of simultaneous possibilities and on the other the possible applications of a word (its locations) should not be static.[187]

There is another way to put the problem that the sign only has sense as part of a system, that this system is determined by the use of the signs, and that therefore it seems that the whole, temporally extended system is somehow compressed in the sign. The above-quoted remark on the illusion that a sign contains its grammar "like a string of pearls in a box" ends as follows:

> [...] The geometrical rules *constitute* the cube (give it a constitution). What I used to say about the 'word-body' is the clear expression of this illusion. (MS114, 48r)

What did Wittgenstein say about the "word-body"? It is again a misleading picture, sometimes also called "meaning-body", which mixes two different kinds of sign. Its motivation was the problem that two signs may have the same appearance and yet be used in very different ways, and hence have different grammars. For example, the word "is" can be used to express predication or equality ("the rose is red" or "two times two is four"). The two words may be conceived as "bodies" that have one surface in common – hence their similar appearance –, but behind that surface differ in their three-dimensional structure. Think of a triangular prism and a tetrahedron (cf. MS110, 112 f.). The word "body" is here to be understood geometrically. In German, *Körper* can mean a physical body, but also a geometrical concept

187. Like Engelmann, Uffelmann stresses that the later occurrences of "grammar" are interchangeable with "use" (Uffelmann 2018, 41). While this is indeed a significant shift of emphasis, I do not think that grammar has lost its formal role completely. Rather, it oscillates between the formal sense and the empirical (use) sense as I shall show below in section 6.2.

(e.g. *Platonische Körper*). This ambivalence is part of the source of confusion. For in its physical sense it seems to suggest that there really *is* a difference to the words in any given moment, namely that they do have these different possibilities of application, symbolised by their different meaning bodies. Another illustration of this problem is the difference of the musical tones #e and f which are identical tones, auditive "signs", but belong to different scales, different "grammars" as musicians indeed say. Wittgenstein acknowledges the temptation to believe that the two tones do sound different, if one thinks about the differing grammatical structures (cf. MS157b, 1v). But the grammar, the system to which the sign belongs, is not contained in the sign; the sign is constituted by the system.

In the *Investigations*, there are more illustrative examples: does he who knows the multiplication table for the numbers 1 to 10 (in German: das *Einmaleins*) think of the whole table; or does he who knows the alphabet think of the whole series of letters in each moment (cf. §148)? The "cube" from the middle-period returns at the beginning of the rule-following considerations where the problem is framed in a very explicit way:

> When someone says the word "cube" to me, for example, I know what it means. But can the whole *use* of the word come before my mind, when I *understand* it in this way?
>
> Yes; but on the other hand, isn't the meaning of the word also determined by this use? And can these ways of determining meaning conflict? Can what we grasp *at a stroke* agree with a use, fit or fail to fit it? And how can what is present to us in an instant, what comes before our mind in an instant, fit a use?
>
> What really comes before our mind when we *understand* a word? – Isn't it something like a picture? Can't it *be* a picture? (§139)

The point of the argument is to show that there may well be a picture before one's mind, but it cannot serve as the justification of one's actions. The picture of a cube may be interpreted as a prism by using a different method of projection. And even if the method of projection was somehow indicated in the mental picture, by adding beams of projection to it, *this* could again be interpreted in different ways.[188] The upshot of these remarks is that there is something like "a *normal* case and abnormal cases" (§142), there are more or less established frameworks in human societies and that something is "right" means just: that someone acts in accordance with an established space. However, this normal usage, too, does not determine all applications of a rule rigorously as the space metaphor suggests. This misleading picture was present in the *Tractatus* where the "proposition reaches through

188. "If the method of projection is a bridge, it is one which is not build until the application has been made" (MS116, 124).

the whole logical space" and is discussed as a problem in the context of rule-following in a manuscript from 1944. "The transitions are actually already made" means I no longer have a choice.

> Once the rule is stamped with a certain meaning it draws the line of how it is to be followed through the whole space [...] No, my description only had sense if it was meant symbolically. – 'S*o it seems to me'*, I should have said. (MS128, 45)

Hypostatising this space in which all steps of the rule are already determined is, again, turning a concept that was used as a tool – a tool to highlight the difference between causal and logical dependency – into a "mythological description" (ibid. 46), in the same sense as a notation can become a mythology when it is taken as the description of *de re* necessities (cf. 5.2.2).

In the *Investigations*, this problem leads, after a detour on the nature of understanding and reading, to the much more intricate debates about the logic of rule-following which I cannot go into here in more detail, and to the social aspects of conceptual frameworks to the concept of normality (about which I will say a bit in 6.2). My aim here was to raise awareness to the fact that this famous group of arguments, the build-up to, and part of, the rule-following passages is in fact another way in which the space metaphor is challenged and that it is related with the break-in of time into the spatial frameworks. It seems as if the rule determines all its applications and implications like a geometry determines all its possible locations and constructions. And these rules appear, like geometries, as expressions of *necessity*, they are "*a matter of course*. As much as it is a matter of course for me to call this colour blue"[189] (PI, §238). However, when the perspective of time and actions is considered, the intuitive assumption becomes problematic.

That the problem is not really settled with the end of the "rule-following chapter" – if it can be isolated like this at all, which I think it cannot – may be seen by means of a remark from 1949, published in the so-called part II of the *Investigations*, that formulates the basic problem again:

> If someone says, "When I heard this word, it meant ... to me", he is referring to a point in time and to a way of using the word. (Of course, it is this combination that we fail to grasp.) (PPF, §7)

189. Compare this to the reference to relativity in which Wittgenstein says that the certainty to call something red is "at the basis of my language-game" (MS164, 82) – here the emphasis is on the constitutive nature of grammar (which is compared to Einstein's move in relativity theory); in the *Investigations* the same certainty explains the misleading assumption that a rule produces all its implications in advance (cf. section 4.5).

6.2 Ambivalences: Logic and Experience

I have already indicated that Wittgenstein's claim to be merely *descriptive* can be understood in different ways. There is the basic opposition between *description* and *explanation* where a clear preference for the former is recognisable. It is based on the rejection of causal theories which is a constant in Wittgenstein's thinking: in the *Tractatus* he said that "belief in the causal nexus" is (the) "*superstition*"[190] (TLP, 5.1361) and in the *Investigations* it is reaffirmed that "it was correct that our considerations must not be scientific ones [...] all *explanation* must disappear, and description alone must take its place" (PI, §109). While description seems to be a rather straightforward activity as opposed to explanation, it itself can become problematic in certain contexts. If it is used in contrast to other concepts than explanation, it is no longer so clear whether Wittgenstein is always on the descriptive side; it seems that he at least acknowledges other aspects of his philosophy which are not descriptive in a strictly neutral way. In the first sub-section, I shall look at a framework's different status whether it is conceived in a *prescriptive* or in a *descriptive* way. In the second, I shall elaborate on an ambiguity I have already mentioned several times: sometimes it seems that by describing a framework an existing feature of reality is described, and sometimes that the framework first needs to be constructed and that instead of "mere description" there is an element of creativity in it.

6.2.1 Prescriptive vs. Descriptive Grammar

To distinguish the two aspects of Wittgenstein's grammatical frameworks, one can use a distinction that is known from the discipline of linguistics: that between prescriptive and descriptive grammar. By prescriptive I mean that the framework of rules is conceived as the range of possibilities in a binding sense. This means that we cannot speak of something that does not fall within this range. To use a word in a way that deviates from common usage, would be to transgress the "bounds of sense", it would be nonsense. By contrast it would be descriptive to simply state how a word is normally used. Since this use amounts to a complicated language-game that is played with the word, it describes a range of possibilities too, the word's possible applications, but the emphasis in this case is on simply *stating* these possibilities rather than on *licensing* them or *legitimising* them as opposed to other *illegitimate* usages. Moreover, the claim of a framework described in this way is not absolute:

190. The contrast between superstition and mistake, which is highlighted in the *Investigations* (cf. §110), corresponds to the distinction between space and place: while a mistake refers to a false location in a space, superstition refers to a space as a whole – the whole way of looking at things is somehow "wrong".

if something unknown occurs, it would simply be recorded and added to the dynamic totality of use instead of being sanctioned as nonsense.

It is not clear on which side Wittgenstein is in this opposition. The *Tractatus* with its logical space for everything meaningful was quite clearly on the prescriptive side. Its purpose was indeed to draw a boundary between sense and nonsense.[191] Parts of this prescriptive element remain vital in Wittgenstein's later thinking. At least in the discussion of colours, the purpose of grammar is clearly to exclude nonsense; likewise in the context of actual games (e.g. castling in draughts). Importantly, it is not only nonsense to violate a rule of grammar, but also to express it as if it were an empirical truth. Let us consider a remark from the "private language argument":

> In what sense are my sensations *private*? – Well, only I can know whether I am really in pain; another person can only surmise it. – In one way this is false, and in another nonsense. [...]

These two ways are the two senses of description we have distinguished: if the statement is a grammatical proposition about a form of expression (knowing that I am in pain is not part of its range of possibilities, not a move in the game) and in this sense prescriptive, then it is nonsense to utter it in the form of a quasi-empirical claim:

> [...] It can't be said of me at all (except perhaps as a joke) that I know I'm in pain. What is it supposed to mean – except perhaps that I am in pain? [...] This much is true: it makes sense to say about other people that they doubt whether I am in pain; but not to say it about myself. (§246)[192]

However, in the other sense, as a neutral description of the use of pain expressions, the statement is simply *false*, because we *do* say that other people know whether I am in pain or not. If someone says about someone displaying pain behaviour "he is in pain" this is not considered a piece of nonsense, but simply an empirical observation – in this case, however, it does not have the metaphysical nimbus of a deep insight which it had when first uttered by the interlocutor. More examples from the *Investigations* likewise imply this prescriptive element of grammar. "When a sentence is called senseless, it is not, as it were, its sense that is senseless. Rather, a combination of words is being excluded from the language, withdrawn from circulation." (PI, §500). If the framework is accepted, it is nonsense to state something that is part of the geometry as if it were a fact. If it is merely a description of the use of a word, then it can always be false as it is in the case of "only I can know my sensations".

191. It was descriptive only insofar as the notation introduced therein was meant to be able to really describe reality.

192. Cf. §401: "if you logically exclude other people's having something, it loses its sense to say that you have it."

That "every description can be conceived as a rule, every rule as description" was already clear to Wittgenstein in 1930 (cf. MS107, 233).[193] A sentence like "this is blue" can mean "this shall be called blue" (in a game for example) in which case it would be a prescriptive and grammatical proposition; or it can be a description in an already established space of things and colours in which an object is said to have the "coordinate" blue – this truly descriptive sentence can always be false, the prescriptive one cannot.

The ambivalence is so confusing because it is one and the same sign which has these different roles. *Qua* grammatical rule a proposition may be timeless and fixed, *qua* description it is valid only at a certain point of time. Conflating the two roles provokes the kinds of statement Wittgenstein argues against constantly: statements taking a formal, prescriptive, feature to be a quasi-empirical fact which has the strange property of being always true. Therefore, his reminders are sometimes meant to highlight the logical, formal, role of such a statement so as to emphasise that something which cannot possibly be false is part of the framework, not a possibility within it; however, when this risks turning into a mythology of metaphysical "super-concepts", he insists that there is also a descriptive aspect to it which makes it fallible just like any other empirical judgement and that, if the use of words and human activities (forms of life) were different, even those seemingly necessary propositions could be different. An awareness of this problem is present at all stages of Wittgenstein's writing of which I have given some evidence in my discussions of spaces and related concepts. But there is a certain development from emphasising the prescriptive side to emphasising the descriptive and changeable side (which has led Engelmann to believe that Wittgenstein abandoned grammar in favour of an "anthropological view"). In the last manuscripts of *On Certainty*, the importance of a statement's role in a specific situation is spelled out most explicitly: certainty is what is taken for granted, what first of all constitutes the possibility of doubt for other things, but which cannot be doubted itself. However, Wittgenstein warns against expecting more than this from our concept of certainty. He warns particularly against the search for a kind of absolute foundation of knowledge, because even those constitutive frameworks that are exempt from doubt can change over time or could be imagined differently, that is, if they are not considered as prescriptive rules in a specific situation, but as historical facts *that* I and other people take this set of propositions for granted at a certain time. This

193. It speaks also against the assumption of a strong break between "middle" and "late" Wittgenstein – for example as "grammar" vs. "use" – that in the context of this remark the importance of use is already very present: he already brings in the metaphor of words as money, not in the sense of reference, but of what one can *do* with them (MS107, 233; cf. also PI, §120). And also the idea of "use in time" is already anticipated here and brought in connection and contrast to that of "the proposition as part of a system of propositions" (MS107, 234).

fluctuation between description and prescription is acknowledged in the *Investigations*,[194] but finds its most illustrative expression in the river-bed metaphor in *On Certainty*:

> It might be imagined that some propositions, of the form of empirical propositions, were hardened and functioned as channels for such empirical propositions as were not hardened but fluid; and that this relation altered with time, in that fluid propositions hardened, and hard ones became fluid.
>
> The mythology may change back into a state of flux, the river-bed of thoughts may shift. But I distinguish between the movement of the waters on the river-bed and the shift of the bed itself; though there is not a sharp division of the one from the other.
>
> But if someone were to say "So logic too is an empirical science" he would be wrong. Yet this is right: the same proposition may get treated at one time as something to test by experience, at another as a rule of testing.
>
> And the bank of that river consists partly of hard rock, subject to no alteration or only to an imperceptible one, partly of sand, which now in one place now in another gets washed away, or deposited. (OC, §§96–99)

To overemphasise the prescriptive aspect of grammar runs the risk of turning into a somewhat authoritarian demeanour. It may then appear as if language was in fact a system (or many) of complicated rules which have to be revealed by the philosopher whose task it is to point out transgressions of those rules: to warn people when they are talking nonsense. Although they show awareness of this problem and occasionally qualify their claims with a *caveat*, scholars like Hacker and Glock tend to highlight rather this side of grammar. They tend to excessively highlight the similarity – which does indeed exist to an extent – between the middle Wittgenstein and the later writings, that is, the sense-constitutive role of grammar as a prescriptive framework (cf. Hacker 2012b) and neglect the shift towards use as implying empirical aspects of logic (and the therapeutic function). Following this prescriptive reading of grammar, they would hold that the "transgression" or "violation" of its rules yields "nonsense" (Baker and Hacker 2009, 19 and 57; cf. Glock 1991, 84–85) and that these rules are rules that everyone *has* to accept.

> A sign becomes meaningful not through being associated with an object, but through having a rule-governed use. Whether a sign is meaningful depends

194. E.g. "What today counts as an observed concomitant of phenomenon A will tomorrow be used to define 'A'" (PI, §79); "what a proposition is, is in one sense determined by the rules of sentence formation (in English, for example), and in another sense by the use of the sign in the language-game" (§136).

> on whether there is an established use, whether it can be employed to perform meaningful linguistic acts; what meaning it has depends on how it can be used. (Glock 1996, 376–368)

> The special function of grammatical reminders is to *draw attention* to the *violation* of linguistic rules by philosophers, a violation that results in nonsense. (Glock 1991, 78, quoted in Kuusela 2008, 242)

A certain tendency is visible in these statements and it has indeed provoked a reaction in Wittgenstein scholarship. Critics of this view have highlighted that this prescriptivism about grammar amounts to a *theory* about language despite the well-known fact that Wittgenstein rejected the view that philosophy should put forward a theory of language (cf. Kuusela 2008; Engelmann 2013b). Engelmann argues that the idea of rule-governed frameworks is more or less replaced by what he calls the "genetic method" and the "anthropological view" which has a focus on facts of language acquisition and use, on the genesis of forms of expression and on their actual application by human beings in an anthropological sense (cf. Engelmann 2013b). Hence, unlike Hacker, he stresses the *differences* between the middle and the late Wittgenstein and even speaks of different philosophies.

While the criticism of the one-sided view of prescriptivism is justified, this view can be blind to the other side and hence miss the ambiguity in Wittgenstein's thinking: it can make it appear as if the prescriptive aspects of grammar have disappeared completely and been replaced by a mere description of human practices for "therapeutic" purposes, to clarify how a certain misleading analogy has arisen without, however, giving a positive account of how things should be looked at.[195]

"The fluctuation of grammar between criteria and symptoms makes it appear as if there were nothing but symptoms" (PI, §354).[196] Beyond the philological evidence that there are indeed passages in the *Investigations* and later that indicate an at least locally prescriptive reading of grammar, there is also a philosophical uneasiness about this purely descriptive view. It threatens to eliminate the difference between empirical and grammatical propositions which is never neglected in Wittgenstein, even when the empirical side of the latter is acknowledged as in *On Certainty* ("But I distinguish between the movement of the water and the river-bed itself"). To say that there is no grammar, that there are no prescriptive rules at

195. Kuusela, who agrees with Engelmann on the rejection of the authoritarian and "theoretical" accounts of Hacker and Glock, recognises this shortcoming of a purely therapeutic approach (cf. Kuusela 2019a).

196. For a detailed discussion of criteria and symptoms see Cavell (1979). It is clear why this remark belongs to the context: criteria are on the level of geometry, they prescriptively say that, if A is a criterion of B, and A is given, then B is true. By contrast, symptoms are "softer" insofar as they only say (descriptively), if A is a symptom of B, and A is given, then B might be true, but needn't be necessarily true.

all, because they can change and can be described in a quasi-empirical manner, is like saying that there is no river since its water is in constant flux and even the river-bed may change. It is the Heraclitean fallacy that something dynamic cannot be talked about as a stable entity at all which Wittgenstein explicitly addresses in the context of time questions quoted above: "The man who said that one couldn't step twice in the same river, said something false. One can step twice into the same river" (MS110, 34). To see that reality is ever changing and fleeting is not to say that we can never speak about reality; we can and do speak about it and if actual language is the reference point then we see that the word "fleeting" itself only makes sense as a concept that is opposed to something stable. Likewise, to see that a grammatical framework can never *a priori* capture the totality of empirical possibilities, is not to say that we can never construct any framework that determines things *a priori* or that we are not in fact guided by such implicit structures when we use language. These are the formal structures whose role in Wittgenstein's considerations I have tried to extract from his writings in the preceding chapters.

6.2.2 Grammatical Realism vs. Constructivism

As I said, Kuusela agrees with the criticism of too much prescriptivism, but warns against a merely descriptive and therapeutic view. He suggests understanding rule-governed frameworks such as calculi and games as constructed models, as objects of comparison, with the aim of clarification. Thereby they are dependent not only on the context, but also on the person who stands in need of clarification. Describing these models is *one* method of philosophical clarification among others.[197]

Here we encounter another problem for the purely descriptive account of philosophy. A purely descriptive account of constitutive frameworks would be to assume that there really is something like a grammatical space which can be described; it is a question of realism about these geometrical structures which Wittgenstein himself seems to be rather sceptical of. However, it remains a recurrent temptation for him to reify expressions of potentiality like "form", "space", "location", or also "moves in a game", "applications of a rule" as if they were something like real objects. To treat the constitutive frameworks as constructed models counters this realism, but in that case they cannot be accounted for with mere descriptions.[198]

This ambivalence is related to the distinction of prescriptive and descriptive grammar: the problem of philosophical authoritarianism becomes pressing if one combines a realist view

197. Another method would, again, be "anthropological": remarks about the natural history of humans (cf. Kuusela 2019b, 182).

198. One could perhaps characterise these positions as idealism and realism which Wittgenstein both rejects: "The one party attacks the normal form of expressions as if they were attacking a statement; the others defend it, as if they were stating facts recognized by every reasonable human being" (PI, §402).

about frameworks with the claim that their objectively existing rules really and bindingly determine what makes sense, what is rational and so on.[199] It is somewhat less threatening if one considers the frameworks as man-made systems which can be altered if required. The non-prescriptive descriptivism is compatible with grammatical realism, but seems to be philosophically empty: since the frameworks are not granted any prescriptive power, to merely describe them, even if they are real, is no more than an anthropological observation – it can have a philosophical function in a negative sense, insofar as it offers an explanation of how certain misleading expressions have come about. To merely describe constructed frameworks is, of course, also possible, but if one does not ascribe any influence to them on questions of sense and nonsense, then the idea of constructing new frameworks seems to lose its point.

I do not want to straightforwardly identify with one of these approaches; naturally, the ambiguity of each distinction is reflected in the ambiguity of their combinations. Neither in the distinction descriptive-constructivist nor in the distinction descriptive-prescriptive is there a clear preference for one side. Hence, I would again subscribe to a polyphonic reading of Wittgenstein in which various voices are presented none of which is declared *the* correct one in an absolute sense. What I find most interesting among the combinations of the distinctions presented is the idea that frameworks are constructed and prescriptive where the non-realism somewhat extenuates the authoritative claim. However, this would completely neglect the descriptive side of both distinctions: that there is also a certain factive element in these frameworks which restricts our freedom to construct them. So in the end one has to accept the ambiguities and highlight one side or the other according to the context of the philosophical problem at hand.

If language-games are only taken as constructed models to illuminate certain points of reality then our freedom is almost unrestricted, but this clashes with Wittgenstein's claim for being purely descriptive which requires something like a factive element. As we have seen, he insists that language need not conform to a calculus or a game, that these are only objects of comparison (cf. PI, §81) – and this seems to imply that they can be constructed. The paragraph opens with a reference to Ramsey who called logic "a normative science". Now one could say that if the propositions of logic (or grammar) are *norms*, then they must have some prescriptive power: unlike factive statements, norms speak about what *ought* to be done, not about what is the case (Glock, for example, who tends to a prescriptive reading of rules, insists on their normative nature). But Wittgenstein does not seem to use "normative"

199. There would then be something like absolute nonsense which completely fails to accord with the form of reality which is reflected in grammar. Hacker's reading of the late Wittgenstein is anti-realist (cf. Hacker 2017) insofar his philosophy is only about forms of language, not about *de re* facts and necessities. However, Hacker is a realist about these grammatical forms: they exist and determine sense and nonsense.

in this sense. That logic is *not factive* has been his basic conviction since the *Tractatus* which he has never doubted anyway. So why should he make that distinction? One could read the remark making almost the opposite claim: if logic is normative, then it is not the reflection of metaphysical necessity, but has contingent, factive, aspects to it. For unlike metaphysical entities, norms can be described, they can change and so on. In a notebook with drafts for the *Investigations*, Wittgenstein seems to refer to Ramsey's view precisely in this vein: "I always wanted to say (against Ramsey): Surely, logic cannot become an empirical science. But how we use the language/words, this is, of course, experience" (MS152, 93–94).

The question returns in the idea of forms of life, the cultural background which functions as a constitutive framework. The emphasis of these latter frameworks is clearly on the empirical, on the factive side of our dichotomy. By taking up a foundational role in language, they are on the one hand constitutive of meaning, a characteristic of the form we have established: human agreement, agreement in form of life, is not agreement in opinions (cf. PI, §241); it determines what *can* be true, not what *is* true in a specific situation. On the other hand, we cannot influence or construct forms of life; in this sense they are a foundation, a number of empirical facts about the way human beings behave. *Lebensform* is more on the use side of our frameworks. A form of life is, as it were, the totality of uses; these, in turn, determine which language-games can be played, what counts as having sense in which situations and so on. In this sense they could be the object of an anthropological study indeed. The difficulty to balance between the factive and the logical side of such systems is expressed in the remark following that on "agreement in forms of life":

> It is not only agreement in definitions, but also (odd as it may sound) agreement in judgements that is required for communication by means of language. This seems to abolish logic, but does not do so. – It is one thing to describe methods of measurement, and another to obtain and state results of measurement. But what we call "measuring" is in part determined by a certain constancy in results of measurement. (PI, §242)

> What you say seems to amount to this, that logic belongs to the natural history of man. And that is not compatible with he hardness of the logical "must".
>
> But the logical "must" is a component part of the propositions of logic, and these are not propositions of human natural history. If what a proposition of logic said was: Human beings agree with one another in such and such ways (and that would be the form of the natural-historical proposition), then its contradictory would say that there is here a *lack* of agreement. Not, that there is an agreement of another kind.

> The agreement of humans that is a proposition of logic is not an agreement in *opinions*, much less in opinions of logic. (MS164, 149–150; cf. RFM, 352–353)

What seems central to me, is not to absolutise either horn of the dilemma: After acknowledging that forms of life are more on the factive side, admitting that even the river-bed may change, Wittgenstein immediately qualifies this insight. The changeability does not amount to equalising the river-bed with the river. Apparently reconsidering Ramsey's idea he writes: "But if someone were to say 'So logic too is an empirical science' he would be wrong. Yet this is right: the same proposition may get treated at one time as something to test by experience, at another as a rule of testing" (OC, §98). In the first sense it is a description of something "real" which can be tested, in the second sense it is constructed. Simply describing human practices in the first sense obviously borders to the discipline of anthropology.[200] However Wittgenstein is aware of the risk of turning philosophy into an empirical science. His "ethnological view", he writes in 1940, "means that we take a point of view far afield so as to see things more *objectively*" (MS162b, 67v). – What was considered impossible in the *Tractatus*, to be outside of logical space, becomes possible in the later philosophy thanks to, first, the plurality of spaces and, second, to the ambiguity of grammar between the logical and the empirical: now there is the possibility to treat one's own constitutive framework like the culture of a foreign people, that is, to simply describe the use of words and the activities with which it is interwoven as if they were not treated as given certainties in one's own culture. However, this objective view will reveal that some propositions *are* used as certainties and do have a foundational function, but this we know not through insight into the necessary form of the world, but by describing it as *one* form out of many possible.

Besides Engelmann and Kuusela, other scholars have noticed this turn towards anthropology. Danièle Moyal-Sharrock (2003) focuses on the foundationalist function of what Wittgenstein calls "hinges" in *On Certainty*.[201] However, the foundational role of a framework – I would subsume "hinges" under my "geometrical" propositions since they are exempt from doubt[202] – is not the absolute one which the foundationalist strives for. "The difficulty

200. For Kuusela, it is an extension of logic as the method of philosophy. I agree by understanding anthropological frameworks, forms of life, as extensions and modifications of spatial frameworks.

201. Besides, she highlights the universal nature of some anthropological facts, but given the preceding discussion of Wittgenstein's anthropological methods, I do not think he would be interested in such a contingent universality. The point of seeing something sub specie anthropology is precisely to imagine that seemingly necessary things could be different.

202. It is debatable whether they are "grammatical": for Hacker, only propositions governing the use of language are grammatical, hence hinge propositions are not, because they are of the form of empirical propositions (cf. Baker and Hacker 2009, 258; cf. Bassols 2010). However, Moyal-Sharrock rightly highlights that they *seem* to be empirical, but function like logical statements to which a certain necessity is ascribed. I would agree with her and say that they belong to the framework of fixed certainties which first of all constitute the possibility of empirical truth and falsity.

to do without any theory is this: to take what is incomplete as something complete" (MS133, 73v–74r). Instead of treating something as necessary because it is subjectively necessary for our activities, the ethnological viewpoint is to see these things as something a certain people holds necessary at a certain time. This "anthropological, or even anthropomorphic, view of necessity [...] may be disappointing", as Stanley Cavell has pointed out. It seems to be not the kind of thing we had in mind when we talked about logic (it seems to "abolish logic"): "as if it is not really *necessity* which he has given an anthropological view of. As though if the a priori has a history it cannot really be the a priori in question" (Cavell 1979, 118–119). But the point is, despite all these historical and factive elements, to still recognise its grammatical or logical role in a specific situation – its role as an *a priori* framework which determines what counts as possible.

In my view, the role of anthropology or "natural history" is as related with the idea of frameworks as are the spaces, calculi and language-games of the middle and late period; only they stress the empirical side of these frameworks, the one which is real and can be described, not the constructivist side which they also have. Based on my study of spaces so far, I would say that the former aims at the clarification of the space we are placed in, it may be an inherited framework or conventions, and that the latter aims at inventing new spaces – and that both aspects are valid and can help to avoid confusion in our attempts to capture reality with our systematic frameworks. Indeed a description of reality without any framework is not possible because the space first of all establishes the means to describe anything, it determines sense, and we cannot do without sense, even if none of these sense-constitutive frameworks is *the* real one. "There is no outside; outside we cannot breathe" (PI, §103).

An indication of the creative aspect of framework construction is visible in Wittgenstein's views on mathematics which is often conceived in terms of spaces (see section 4.3), notations and calculi. On the one hand their seemingly metaphysical and eternal truth can be demystified by pointing out that it depends on actual practices and can be observed as part of our natural history, as a cultural practice

> [...] I go through a proof and then accept its result. – I mean: this is simply what we *do*. This is use and custom among us, or a fact of our natural history. (RFM, 61; cf. MS117, 84–85)

> But isn't it correct to say: The *essential* thing about mathematics is that it forms concepts? – For mathematics is after all an anthropological phenomenon. Thus we can recognize it as the essential thing about a great part of mathematics (of what is called "mathematics") and yet say that it plays no part in other regions. (RFM, 399; cf. MS124, 115)

However, this anthropological perspective also reveals that mathematical truths have a different *role* than empirical truths, are treated as *a priori*, eternal, necessary and so on. The two senses of description are to be considered here: any proposition *could* be considered a mere description of a state of affairs; but given the use of a mathematical, or any prescriptive, proposition, it turns out that they are rather to be understood in the sense of the description of grammar, of a rule, not of a thing.

> Are the propositions of mathematics anthropological propositions saying how we men infer and calculate? – Is a statute book a work of anthropology telling how the people of this nation deal with a thief etc.? – Could it be said: „The judge looks up a book about anthropology and thereupon sentences the thief to a term of imprisonment“? Well, the judge does not USE the statute book as a manual of anthropology. (RFM, 192; cf. MS117, 172)
>
> We shall see contradiction in a quite different light if we look at its occurrence and its consequences as it were anthropologically – and when we look at it with a mathematician‘s exasperation. That is to say, we shall look at it differently, if we try merely to describe how the contradiction influences language-games, and if we look at it from the point of view of the mathematical law-giver. (RFM, 220; cf. MS117, 256)

The development of these *a priori* frameworks is for Wittgenstein a creative task as we have seen: the mathematician is a law-giver, “not a discoverer, but an inventor”. In a manuscript from 1942 in which the mathematical is particularly intimately intertwined with philosophical questions, he alludes to this kind of spontaneity in inventing new frameworks by using the Kantian terminology of synthetic *a priori* (and: “form”) which has already been used in 1931. “What makes one speak of a *synthetic* proposition is the *new* form” (MS125, 79r).

With regards to the two senses of style that I have distinguished (section 5.4.2), one can say that Wittgenstein’s effort to find the right mode of expression for his thoughts – style in the individual sense – is an effort to change other people’s style of thinking, now in the more general sense of a cultural style as an *a priori* framework. It is often quoted that, for Wittgenstein, philosophical problems, philosophical diseases, result from a “one-sided diet” of examples (PI, §593). Elsewhere he says that this “disease of the philosophical problems can only be cured through a changed mode of thinking and mode of life” (MS121, 27r). And in his space phase he had already articulated the essential idea in similar imagery: “He who teaches philosophy today gives dishes to the other, not because he likes them, but in order to change his taste” (MS112, 223). In this sense, I also read the well-known statement that “philosophy ought actually only to be written like *poetry*” – “*Philosophie dürfte man*

eigentlich nur dichten" (MS146, 25v). His stylistic efforts are attempts to achieve something which poetry occasionally achieves, namely to invent new forms of expression, to see things in a new space and to look at the world differently.

However, the ambivalence discussed in this chapter must not be forgotten. Wittgenstein seems to have seen this as a central problem of his whole philosophy. Now, I think, the remark from 1946, which I have quoted in the introduction, has become more understandable:

> If the mode of representation is arbitrary (*willkürlich*), why do not all humans learn the same language – which would surely be so much more practical?
>
> The forces that determine *how* something is represented are as great as those that insist on the truth of the representation.
>
> How small a thought it takes to fill a whole life! (MS131, 180)

6.3 Summary

In this chapter, I investigated the relation of space and time by means of a comparison of the spatial frameworks in the early and middle Wittgenstein and of geometrical propositions, which were thought to feature a timeless necessity, with the more dynamic frameworks of calculi and games in the late middle and late periods. These do not treat of static locations, but of possible moves and actions. There are not only many grammatical frameworks which constitute sense, as shown in chapters 4 and 5, but these may also change over time, as they themselves are partly constituted by the use of language which is extended in time.

The basic opposition that has taken shape is one between timeless and *a priori* spaces on the one hand and a temporal and empirical reality on the other. Between these two there is a dialectical relationship in which no side makes really sense without the other. Speaking of a continuous readjustment of grammatical spaces stretches the concept of logic, but Wittgenstein acknowledges the logical role of certain propositions that have a certain necessity in a particular situation. Much of his work is concerned with the ambivalences of these propositions: that they can be regarded as prescriptive rules or merely as empirical regularities; as features of reality, insofar as they merely describe the practices of human beings and their use of language, or as constructed models which alter our ways to look at things.

Chapter 7

Conclusion

Aus mehr als einem Grunde wird, was ich hier veröffentliche, sich mit dem berühren, was Andre heute schreiben.

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Philosophische Untersuchungen

We are now in a position to specify the claim that there is a certain unity in Wittgenstein's work and that it can be seen under the perspective of space. Space is just one name for a number of connected concepts that play a central role in his philosophy. They stand for formal systems by means of which philosophical problems are conceived. "Space" is particularly suited to reveal the function of these systems for two reasons. *First*, because its history provides a conceptual basis Wittgenstein could build on. The historical development of space concepts in the 19^{th} century prepared the ground for his use of logical space in the *Tractatus*. In the gradual abstraction and formalisation of geometry since Kant, which I have reconstructed, space was linked to concepts such as "form" and "a priori" which remain important throughout Wittgenstein's work and which are connected to all concepts investigated in this thesis. *Second*, space is particularly suited to represent these systems because of its enormous importance in large parts of Wittgenstein's work. In my reconstruction of the Tractarian system I have shown how it draws together key ideas of the book such as the modal ontology, the picture theory, and the rejection of logical constants; it also embodies one of the work's major mistakes, namely its claim for absoluteness and universality. Spaces remain, or become even more, central in the middle period. With numerous examples, contextualisations, and interpretations, I have shown the variety of their occurrences which correspond to so many forms and methods.

Unlike scholars who draw sharp distinctions between middle and late Wittgenstein, I have argued that essential ideas of the *Investigations* are already in place in the 1930s and are accounted for with the spatial concepts I have presented; – that the problems of this imagery are already recognised in these early writings, supports my claim and allows me to provide an account of the changes from middle to late Wittgenstein without losing sight of the similarities. Having carved out the functions of spaces allowed me to recognise other concepts as related. These other concepts still share with spaces the function to distinguish between a level of necessity – the level of the framework – and a level of contingency, the possibilities inside the framework, one "location" as opposed to others which are *a priori* equally possible. As the language of space and geometry, and eventually also of calculus, becomes less important in Wittgenstein's thinking, the systems lose their technical-mathematical roots and increasingly become general metaphors for various ways to look at things. Sure enough, this was already implied by the former concepts: it is a shift of emphasis which does not alter everything that has been valid about constitutive frameworks.

That the systems, especially if conceived as static spaces, bring problems with them, does not weaken, but confirm my claim for their importance. Much of Wittgenstein's thinking after 1929 is concerned with the articulation of these problems which, however, never lead to a complete abandonment of the idea that formal systems constitute the way we look at things and determine the range of possibilities. The task of the philosopher, according to my reading of Wittgenstein, is to think precisely on this level of spaces.

In this thesis, I have interpreted a number of Wittgenstein's self-reflective remarks about his own method by means of my reading of spaces. The most famous one has not been among them and it seems at first sight to be unrelated to my approach: "What is your aim in philosophy? – To show the fly the way out of the fly-bottle" (PI, §309). And yet in the first version of that remark, Wittgenstein does place it in the context of spaces. In 1937, he is discussing a situation in which someone tries to solve a geometrical puzzle, namely to "match" a triangle and a hexagon, and is later surprised that they can be matched by placing the hexagon *inside* the triangle. It is not that there is some sort of resistance against bringing them in this position, but one simply tries everything to match them except *this*.

> This position is, as it were, excluded from the geometry/space. As if there was a "blind spot" in our mind. – And is it not really like that, when I believe to have tried all *possible* positions, but have always, as if bewitched, missed this one.
>
> Can one not say: the representation that shows you the solution removes a blindness; or also, it changes your geometry. It shows you, as it were, a new dimension of space. (as if one showed a fly the way out of the fly-bottle) (MS118, 44v)

Not much has been said about aspect change in this thesis. In the context of the fly-bottle, it becomes clear how my reading allows for an interpretation of it that does not treat it as a completely new topic in a relatively late manuscript, the so-called part II of the *Investigations* with the famous duck-rabbit head, but as connected and indeed central to Wittgenstein's philosophising. Conceiving of aspect change as a change of space, in the wide sense I have elaborated, allows for a solution of the "paradox" that in aspect change everything stays the same and yet changes in a significant sense. Arif Ahmed distinguishes between the "optic" features of an image, which stay the same, and the "*synoptic* content", which changes. By stressing the synoptic side of aspect change, he highlights the organic nature of the experience of an object which includes relations to other objects, actions, reactions, and context. (cf. Ahmed 2017). As we have seen, it is problematic to conceive of an object in complete isolation, outside of any conceptual framework, but I sympathise with the idea that it is a synoptic, a holistic structure which determines the aspect change. Wittgenstein is not merely interested in the psychological phenomenon, but in the logical issue, in the change of spaces to which a philosopher can contribute by clarifying concepts or inventing new spaces that constitute new possibilities (note the emphasis on "possible" in the quote about the fly-bottle).

This is the creative task of the philosopher that I have pointed out in the last chapter: to give dishes to the other "in order to change his taste", to change his "way of looking at things" (cf. PI, §144). That the other also has to accept the "picture" that is thus proposed to him, calls indeed for some skill on the side of the philosopher: the result of clarification or the suggested new framework should be *perspicuous* – to use another key word from Wittgenstein scholarship that has been somewhat neglected in this thesis. This presentational skill as well as the creativity in inventing new spaces is what I take to be at the basis of Wittgenstein's dictum that "philosophy ought actually only to be written like *poetry*". It aims for a change of perspective in the reader or listener that can be described as a change of space, opening up new possibilities, in the strong sense of possibilities that have not been conceivable before. If this change is a change of the rather large frameworks I have discussed, it amounts to the kind of "conversion" that Stanley Cavell regards as an important and decidedly ethical aim of Wittgenstein's philosophy (cf. 1979, 125).

The results of my work reveal many intersections of Wittgenstein's thinking with his contemporaries. Not only does he stand in the tradition of an increasingly abstract and formal account of space and geometry, which enabled his creative use of that imagery for new purposes, but he also has connections to other thinkers who reacted to similar developments at the same time. Michael Friedman has shown how Logical Positivism was influenced by Kant, Poincaré, and Einstein at least as strongly as by Comte and Mach (cf. Friedman

1999). The works of those three thinkers were milestones in the development of space conceptions that Wittgenstein took up and modified in the *Tractatus*. According to Friedman, philosophers such as Reichenbach and Carnap developed an account of *a priori* principles that can be called "relativized a priori": it assumes constitutive frameworks without the claim for apodicticity that characterised the Kantian *a priori*. Wittgenstein's grammatical spaces and related concepts can be conceived as versions of this idea, insofar as they constitute a certain domain without claiming absolute and timeless validity.[203] However, his thinking is more pluralist – "There are many more language-games than Carnap and others can dream of" (MS134, 120) – and more open than Friedman's rather unifying and reductionist account of *a priori* frameworks which may be "relativized", but whose succession is assumed to converge to an ideal of rationality as conceived by mathematical physics.[204]

As we have seen, references to science and measurement as well as to particular scientists abound in Wittgenstein's work. My approach can help appreciating it as a contribution to (among other things) the philosophy of science –and surprisingly for a reason for which he is often considered an enemy of science in general: he insisted on the difference of philosophy and science, of conceptual questions on the level of spaces and empirical questions inside a given space. Another thinker who, according to Friedman, uses a form of the "relativized a priori" is Thomas Kuhn. The relation between his theory of paradigms and Wittgenstein's grammatical frameworks may be even closer than is generally assumed. Kuhn was influenced by Ludwik Fleck, a contemporary of Wittgenstein who had a conception of "styles" that is strikingly similar to what I have discussed in section 5.4.2, even though they probably did not know each other. While there are studies about Wittgenstein's relation to these thinkers (for Fleck see Griesecke 2008; Kogge 2008; for Kuhn see Kindi 2017), my approach allows for a more systematic comparison on the basis of frameworks that constitute what counts as possible in the first place. Fleck argues that styles of thinking determine what is considered meaningful by a group of thinkers. When Wittgenstein made his claim that he had developed a new philosophical method in 1933, he said that "the style of thinking has changed" (MN, 67), referring to the German tradition which understands style in the sense of an *a priori* framework (cf. Plaud 2011, 76). I argue that this method is not replaced later by a plurality of methods, but that it is an essentially open method which acknowledges the need to treat different forms differently. The method to conceive of logic in terms of systematic frameworks, is not abandoned, but transformed and put into dialogue with ever new claims of an unsystematic reality.

203. In the debate on Friedman's ideas, Wittgenstein is mostly considered a "logical absolutist" because only the *Tractatus* is considered. See my paper *Beyond logical absolutism. Wittgenstein and the relativized a priori (forthcoming)*.

204. For criticisms of Friedman's view in this vein see Chang 2008; Mormann 2012; Stump 2015.

Finally, I want to come back to the idea of philosophy as poetry. The epigraphs that I used for the individual chapters in this thesis are taken from texts that were written at Wittgenstein's time. While Spengler's book was published in 1918 and Adorno's late work sums up thoughts from the 30s to the 60s, Robert Musil and Hermann Broch coincide with the middle Wittgenstein not only temporally – their great novels were published from 1930 to 1933 – but also locally as they were based in Vienna for much of that period. Their thinking in this period is primarily concerned with the question of formal systems that determine how a given situation is conceived and how such a space affects our actions. Based on my understanding of constitutive frameworks, it would be possible to investigate from a fresh angle these similarities between Wittgenstein and writers who worked, as it were, at the edge of the Vienna Circle and added an aesthetical dimension to its formalist methods.

In light of all these connections, which come into light through a focus on frameworks, one may revise a common cliché about Wittgenstein. Even though I consider thinking in "spaces" as a recognisable characteristic, "Wittgensteinian" philosophy may after all not be as idiosyncratic as it is often presented.

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